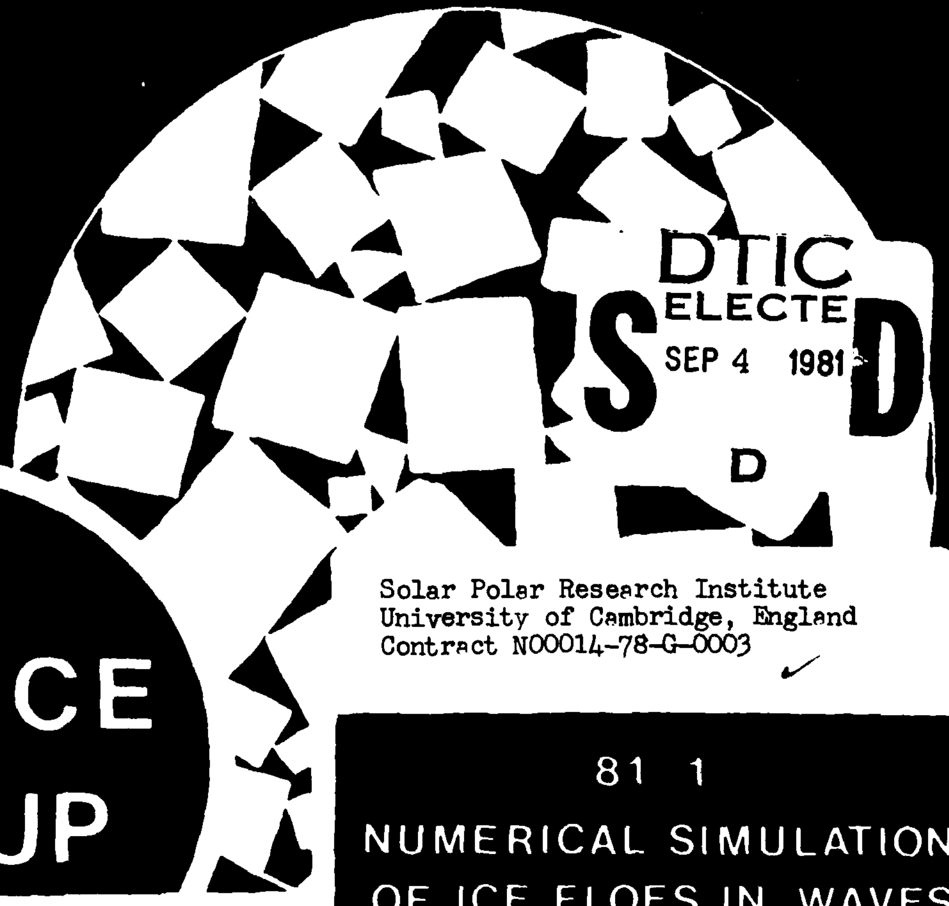
AD-A103 794 SCOTT POLAR RESEARCH INST CAMBRIDGE (ENGLAND) F/G 8/12
NUMERICAL SIMULATION OF ICE FLOES IN WAVES.(U)
1981 V A SQUIRE N00014-78-G-0003
UNCLASSIFIED SPRI-TR-81-1 NL

1 OF 1
AD A
103794

END
DATE
FILMED
10-81
DTIC

SPRI



Solar Polar Research Institute
University of Cambridge, England
Contract N00014-78-G-0003

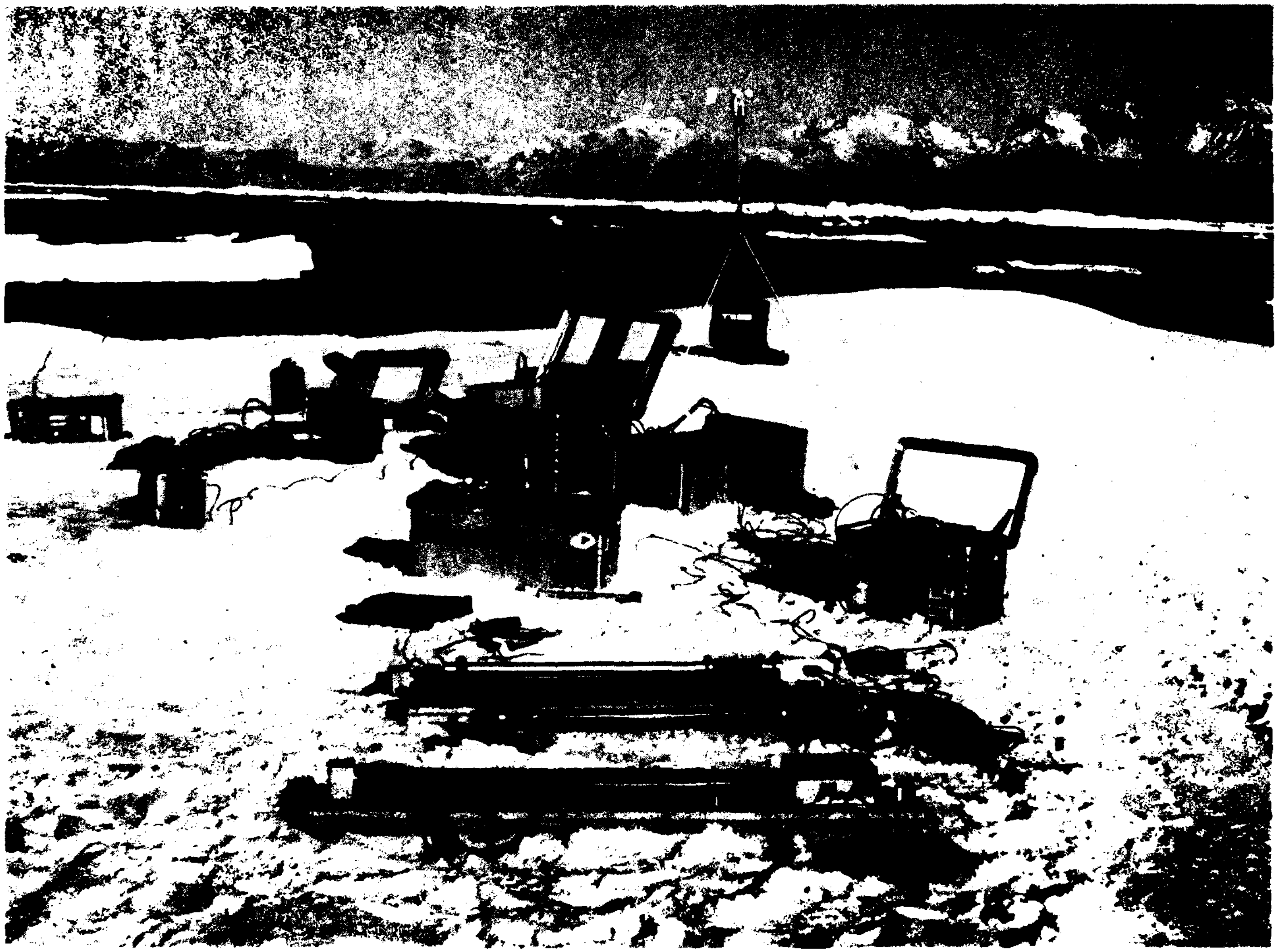
SEA ICE GROUP

81 1

NUMERICAL SIMULATION OF ICE FLOES IN WAVES

Vernon A. Squire

1981



Frontispiece. Experiments in east Greenland during summer 1979.

NUMERICAL SIMULATION OF ICE FLOES IN WAVES

By

Vernon A. Squire
Sea Ice Group
Scott Polar Research Institute
University of Cambridge
England

SCOTT POLAR RESEARCH INSTITUTE
TECHNICAL REPORT 81-1

SPRI-TR-81-1

ABSTRACT

A complex numerical model to compute the behaviour of an arbitrarily shaped ice floe in ocean waves is proposed and discussed. The model has two independent and decoupled phases: firstly the motions and pressures are found for a totally rigid ice floe; and secondly the assumption of rigidity is relaxed so that the flexure may be computed from the under-ice pressure loading. Finite element techniques are used in the latter calculation so as to retain maximum generality. A detailed discussion of the method is given with special attention paid to the problems encountered in its development. The model is demonstrated in both a static and dynamic sense for an ice floe of simple cross-sectional shape in waves of various period.

Although no detailed comparison with recorded wave data can be carried out at this time, a brief section discussing the application of the model to east Greenland data is included.

ACKNOWLEDGEMENTS

I am indepted to Prof. Choung Lee of the Naval Ship Research and Development Center in the United States for his dedication when implementing the ship-motions program on the Cambridge computer. I also wish to thank Dick Henshell and Neil Rigby of PAFEC whose guidance enabled the finite element model to be applied in a somewhat unconventional manner. The fieldwork reported in this article took place near Mestersvig in East Greenland, and I am grateful to Peter Wadhams and other members of the Sea Ice Group for their efforts in making it a successful field programme. Finally I thank Dougal Goodman and Monica Kristensen for helpful and critical discussion, and Rob Massom for his drafting skills.

This work was supported financially under Office of Naval Research contract N00014-78-G-0003 and by the British Petroleum Co., Ltd. The Natural Environmental Research Council of Great Britain provided salary support for the author.

TABLE OF CONTENTS

	PAGE
1. INTRODUCTION AND DISCUSSION	1
2. THE MOTIONS OF ICE FLOES IN WAVES	4
2.1 Introduction	4
2.2 Mathematical Foundation of the Model	4
2.3 Simulation of the Motions of an Ice Floe	8
3. FLEXURAL BEHAVIOUR OF ICE FLOES IN WAVES	17
3.1 Introduction	17
3.2 The Sub-Ice Pressure Field	20
3.3 Preliminary Discussion of the Finite Element Model	21
3.4 Application of the Method	26
3.5 Dynamic Analysis	38
4. PRELIMINARY COMPARISON OF THEORY WITH EAST GREENLAND DATA	47
4.1 Introduction	47
4.2 Results of Comparison	49
5. BIBLIOGRAPHY	55

1. INTRODUCTION AND DISCUSSION

Since 1959 when Gordon Robin used a ship-borne wave recorder aboard RRS John Biscoe to measure waves in the Antarctic (Robin, 1963), the Scott Polar Research Institute has been involved in studies relating to the interaction of ocean waves with pack ice. A series of papers and reports have been published which have used novel techniques for studying penetration of waves into fields of ice floes, and which have posed such questions as: a) what happens to the open ocean energy spectrum as it travels through pack ice; b) what effect do waves have on a particular distribution and concentration of ice floes; c) how does an individual ice floe behave when subjected to ocean waves? These questions have led to several in situ experimental programs which have taken place off Newfoundland, in the Bering Sea, and in the Greenland Sea (see for example Wadhams, 1973a, 1973b, 1978; Squire and Moore, 1980; Squire and Martin, 1980; Moore and Wadhams, 1981). This work has been almost totally experimental except for the model originally proposed by Wadhams in his 1973 Ph.D. dissertation. Using a method devised by Hendrickson (1966), with corrections for sign errors, Wadhams (1973b) approximated the ice floe as a thin elastic plate on deep water. By using a potential matching method, he was then able to find approximate solutions for the waves beneath and on either side of the floe. Unfortunately, the matching could not be carried out perfectly and as Wadhams points out, there are additional unknown potentials in each region which must be included to form the complete solution. The derivation of these potentials represents a formidable task and no exact solution has been found to date even for the linearised equations. More recent work (Squire 1978) has concentrated on another weakness of the early studies, namely in the modelling of the material properties of sea ice, rather than on the lack of sophistication in the hydrodynamics.

Clearly, to be able to answer questions a) and b) above one must first be able to answer question c) since the behaviour of the component ice floes must control the dynamics of the complete ice field. The matched potential approximation is inadequate if one wishes to include all the motions (including bending) of the individual ice floes, but nonetheless has provided a reasonable fit to experimental data collected over the ice pack as a whole. Current interest has, however, been centred around the determination of the characteristics of motion and flexure of a solitary ice floe in waves. For this problem, the matched potential approximation is grossly in error and predicts unrealistic values for the wave generated loads and motions. Two consecutive field seasons in east Greenland waters have been carried out with the prime objective of measuring the dynamics and flexing of discrete ice floes under wave loading. Surface strain, heave and sway were measured aboard several floes of different shape and dimensions, and the waves in the open water near the floes were recorded during the experiment. The two seasons were both extremely successful and have produced a data base which has been analysed during the last year to produce power spectra, frequency response functions between forcing and response, etc. The results have prompted the present study, whereby a theoretical model which is able to compute all the motions of an ice floe as well as its flexural behaviour without the approximations involved in potential matching, has been developed. The model is numerical throughout due to the complexity of the mathematics.

During the analysis of the east Greenland dataset, the author carried out an extensive survey of the literature relating to the mathematics and physics involved in the interaction of waves with sea ice. The problem for an individual floe is not unlike that encountered by naval engineers who are interested in the motions and bending of ships at sea. The modern history of this subject began with Ursell (1949a), who formulated and solved analytically the complex mathematics which arise when a semi-immersed circular cylinder is allowed to move sinusoidally in a fluid. Subsequent work generalised Ursell's cross-section to shapes of particular interest to ship designers. Much more recently, the problem has been reformulated so as to be suitable for solution by numerical methods. In principle this enables all the motions of a body of arbitrary shape in waves to be computed, so that from the naval engineers viewpoint the problem is essentially solved.

Despite the obvious similarities between our problem and that of ship design and engineering, there are very important differences. A naval engineer asks different questions and requires accurate estimates of particular parameters which are not necessarily of interest in the present study. Furthermore, the mechanical properties of sea ice and steel are a world apart so that whereas the rigid body approach may well suffice for a mathematical model of a ship in waves, it certainly would not suffice for an ice floe. For this reason very little work of suitable application could be found on wave induced flexure of ships, so that the author chose to take a novel approach which still allowed any cross-section shape to be analysed.

In this report we will lay down the framework of a method which can be used to compute both the wave-induced motions and the bending for any floating object in waves. The discussion is naturally directed towards calculating the response for ice floes since this was the problem originally posed. The model is equally applicable to any floating or submerged body in waves however, so that the method is far more general than this work indicates. In addition to our ice floe studies we have already been able to use the model to compute the motions of a large tabular iceberg. The calculations show resonances in heave, roll and strain which are also observed in field data obtained during a recent Antarctic field season.

2. THE MOTIONS OF ICE FLOES IN WAVES

2.1 Introduction

The motions experienced by a floating ice floe or berg in waves are the rectilinear motions of heave, surge and sway, and the rotational motions of pitch, roll and yaw as shown in fig. 2.1 (after McCormick, 1973). The waves would also tend to bend the ice as they passed beneath, but for the purpose of the present discussion we will make the plausible assumption that the flexural character of the floe is negligible in comparison to the large "rigid body" motions it experiences. The flexural problem will be discussed in a later section.

In order to solve the six equations which result from the motions, it is first necessary to make a series of simplifications. We assume that the water in which the ice is afloat, is inviscid, incompressible and irrotational so that a velocity potential satisfying Laplace's equation exists. Further, we assume that the motion amplitudes and velocities in the waves are small enough that the equations of motion may be linearised. Finally we two-dimensionalise our analysis so that we only consider the motion of a section of the ice floe (this simplification leads naturally to the so called strip theory of naval engineering, see section 3.4). Subject to these assumptions, the problem and its boundary conditions are well-posed and in principle may be formulated and solved.

2.2 Mathematical Foundation of the Model

The complete solution of the floating body problem may be written down in terms of three separate Laplacian potentials representing incoming waves, diffracted waves, and waves generated by the body's motions. These potentials may, since we have adopted a linearised approach, be added

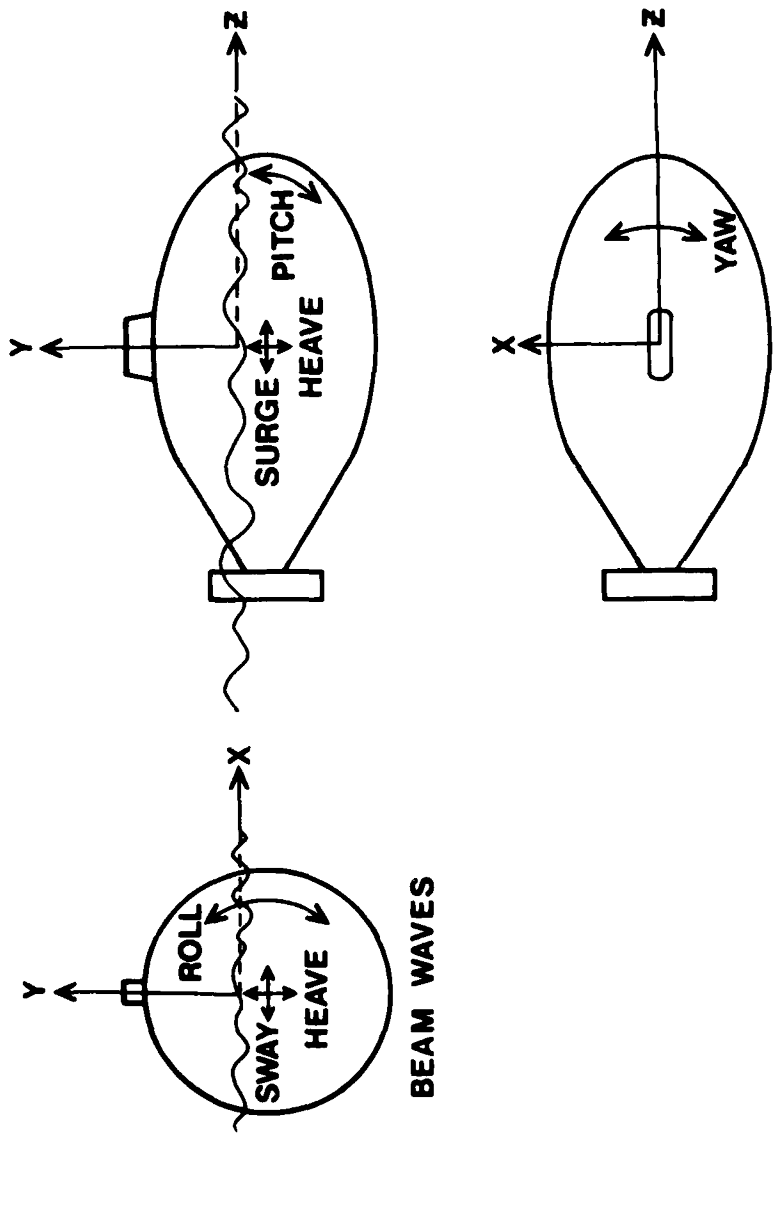


Fig. 2.1 The motions of a floating rigid body in waves.

together to create the velocity potential which represents the total fluid disturbance. Many of the analytic solutions in the literature are concerned with the evaluation of the wave-making potential since in laboratory experiments one wishes to know exactly how a particular wave flume behaves hydrodynamically (see for example Ursell, 1949a, 1949b). The difficulty arises when the complete potential for the floating body problem is required, and various other simplifications have been employed by naval engineers for the motion of ships in waves.

When a body undergoes motion within a fluid it behaves as though it has increased inertia or mass (Lamb, 1962). This is a hydrodynamical effect known as added mass which for cyclic motions will be frequency-dependent. Furthermore, the oscillating body will use energy in creating and maintaining the outgoing waves it generates. This leads to a frequency-dependent damping force in our system of equations. The evaluation of both added mass and damping for arbitrary shaped bodies, at various incident wave periods, is extremely complex. Also the calculation of the diffracted wave potential poses difficulties for all but the simplest of geometries. There is motivation, therefore, for simplifying the equations of motion by neglecting the above quantities. A selection of possible approximations which could be used to model the motion of ships in waves are discussed in detail by Lee (1976). Lee choses to consider five possible cases: a) added mass, damping and diffraction are neglected; b) damping and diffraction are neglected and the added mass is set at the displaced fluid mass; c) as previous case except that diffraction is included; d) added mass and damping are treated correctly but diffraction is neglected; e) complete theory. The assumption that diffraction effects may be omitted is sometimes known as the Froude-Krylov hypothesis, and is equivalent to the supposition that the motions of the body do not alter the particle motions of the fluid although the particle motions influence the body. Lee carried out the analysis for both floating and submerged bodies, and found in the former case that the inclusion of frequency-dependent added mass and damping were essential, and that with no diffraction the computed results (for the sections he chose) were between 20% and 30% in error. For ice floes in waves therefore, it is important that no such approximations be made.

There are two procedures which have commonly been used to solve the floating body problem. The first of these, known as the multipole expansion method, was originally developed by Ursell (1949a) for circular cylinders, and was later applied to sections of arbitrary shape using conformal mapping (the Theodorsen transformation). The second, and more recent technique, represents the problem by a Fredholm integral equation of the second kind, and then approximates that equation by a system of linear algebraic equations. Other procedures exist and are described in detail in an excellent review by Wehausen (1971). The present author's work utilizes a computer program based on the second method.

The program to model floating bodies in waves was originally developed by Frank (1967). Frank approximated the body's cross-sectional contour with a number of nodal points joined by straight line segments. He then used the complex source potential of Wehausen and Laitone (1960), subject to an additional velocity boundary condition across the cylinder's surface, to derive a pair of integral equations. By assuming that the source strength varied from segment to segment but was constant along each segment, Frank was able to write down a set of linear algebraic equations which could be solved numerically. The advantage of this method over the multipole method is that many terms are required in the Theodorsen mapping to treat bodies of non-simple cross-sectional shape. Initial problems concerning wave periods at which no solution to the integral equation exists (John, 1950) and bodies of unsuitable section, have now been overcome (C.M. Lee, personal communication, 1980).

The modified version of the program originally developed by Frank has been implemented on the IBM 370/165 computer at the University of Cambridge. Prof. C.M. Lee of the Naval Ship Research and Development Center in Washington D.C., and the present author encoded three versions of the program; one to derive reflection/transmission characteristics of symmetric ice floes, one to compute the motions of an asymmetric body, and one which included a viscous roll damping which was particularly directed towards iceberg problems. The latter two programs use extended precision arithmetic but still typically run within a few seconds CPU time per wave frequency. The program has been tested many times for various cross-sectional shapes and compares well with experimental results (see for example Frank, 1967; Lee, 1976). With this in mind the author does not propose to repeat these tests, instead we will run the program for an ice floe of realistic dimensions in waves of typical period, and will qualitatively discuss the results by comparing with laboratory experiments in the literature.

2.3 Simulation of the Motions of an Ice Floe

In this, and in subsequent sections, we shall consider an ice floe of typical east Greenland dimensions, viz. 20m beam by 3m thickness. The ice floe will be assumed to be of simple rectangular shape with a density of 922.5 Kg m^{-3}. Neither the shape nor the dimensions are significant and any ice floe whose cross-section is a simply connected region and which could adequately be represented by Frank's polygonal approximation, could be modelled. The ice floe is now assumed to be acted apon by waves between 1s and 20s period. The amplitude of these waves is normalised to 1m so that any plots of heave, sway or roll magnitude may be thought of as the amplitude of the frequency response function for the floe, and plots of pressure are specific to that amplitude. Furthermore, the incident waves are assumed to be beam-on to the ice floe in all cases so that only heave, sway and roll motions will be non-zero.

We begin in fig. 2.2 by plotting the added masses for heave and sway, and the added moment of inertia for roll. The added masses have been non-dimensionalised by division by the mass of the fluid displaced by the floe, and the added moment of inertia with respect to that mass multiplied by the radius of gyration about the centre of roll. Both the added mass and the added moment of inertia curves compare well with those computed by Vugts (1968a) for various cross-sectional contours. There is the same characteristic increase in added mass for heave as period increases, and the same minimum at some point defined by the body's shape, indicating as we suspected, that any theory which neglects the dependence of added mass on frequency would be unacceptable. The added mass for sway and the added moment of inertia for roll show little dependence on period except for short period waves.

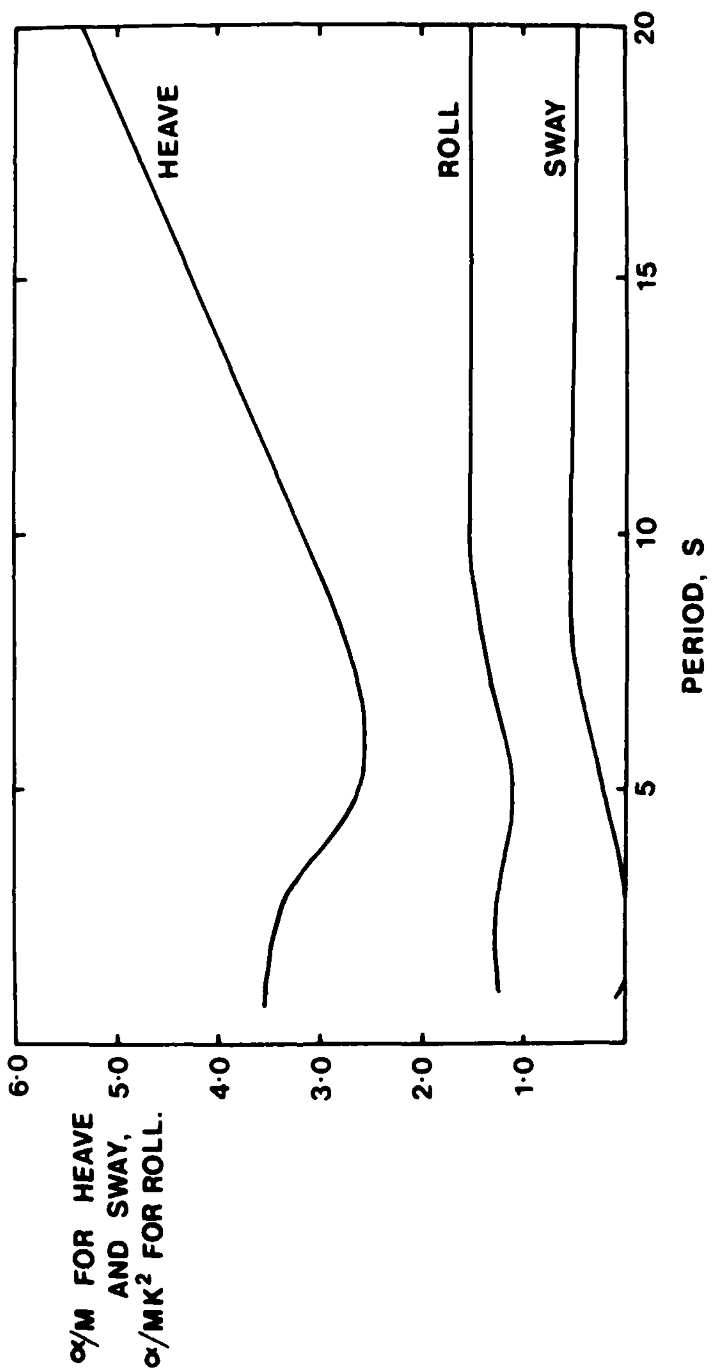


Fig. 2.2 The non-dimensionalised added masses and added moments of inertia for a 20m by 3m ice floe in beam waves.

The various damping coefficients plotted in fig. 2.3 have been non-dimensionalised in a similar way to the added mass curves, with the addition of an extra factor representing the radian frequency of the incoming waves. These plots may also be compared qualitatively with those found in Vugt's paper. The same characteristic maxima in the sway and roll curves are apparent, but Vugt's heave curves have a maximum. On closer examination we find that the dependence of heave damping on frequency is considerably influenced by the draft/thickness ratio of the floating body; as draft increases so the maximum decreases and moves towards increasing period. None of Vugt's cross-sectional shapes have similar draft ratios so are inappropriate for comparison. A close representation of our ice floe is to be found in Frank (1967) where the heave damping ratio is presented for a rectangular section with rounded corners and significant draft. Frank's curve shows exactly the same behaviour as that in fig. 2.3.

Having computed the added mass and damping coefficients as a function of period, we are now in a position to evaluate the wave amplitude ratios due to heave, sway and roll (fig. 2.4). Each of these curves shows a distinct resonance which occurs at a wave period which is quite likely to be found in a open ocean wave spectrum, particularly for an under-developed sea. At a resonant period, the body would tune to the waves so as to produce an enhanced heave, sway or roll magnitude. For long waves, as one might expect, the heave and sway amplitudes tend to unity indicating that the body is simply following the particle motions at the wave surface. Likewise, the roll magnitude becomes small. As one decreases in period, so the response of each motion becomes more complicated: the heave and roll resonate and then decrease so that for waves of less than about 4s periods these motions will not be excited; and the sway magnitude decreases to a minimum at about 7s, resonates at around 5s and then also decreases. The complex shape of the sway curve is interesting since it means that for our ice floe one would expect to see enhanced or suppressed component motions at certain wave periods. The shape of the curves is similar to those measured by Vugts (1968b).

An interesting effect described in that paper is found when the phase difference between roll and sway is computed as a function of wave period. This calculation has been carried out and is shown in fig. 2.5. At a period

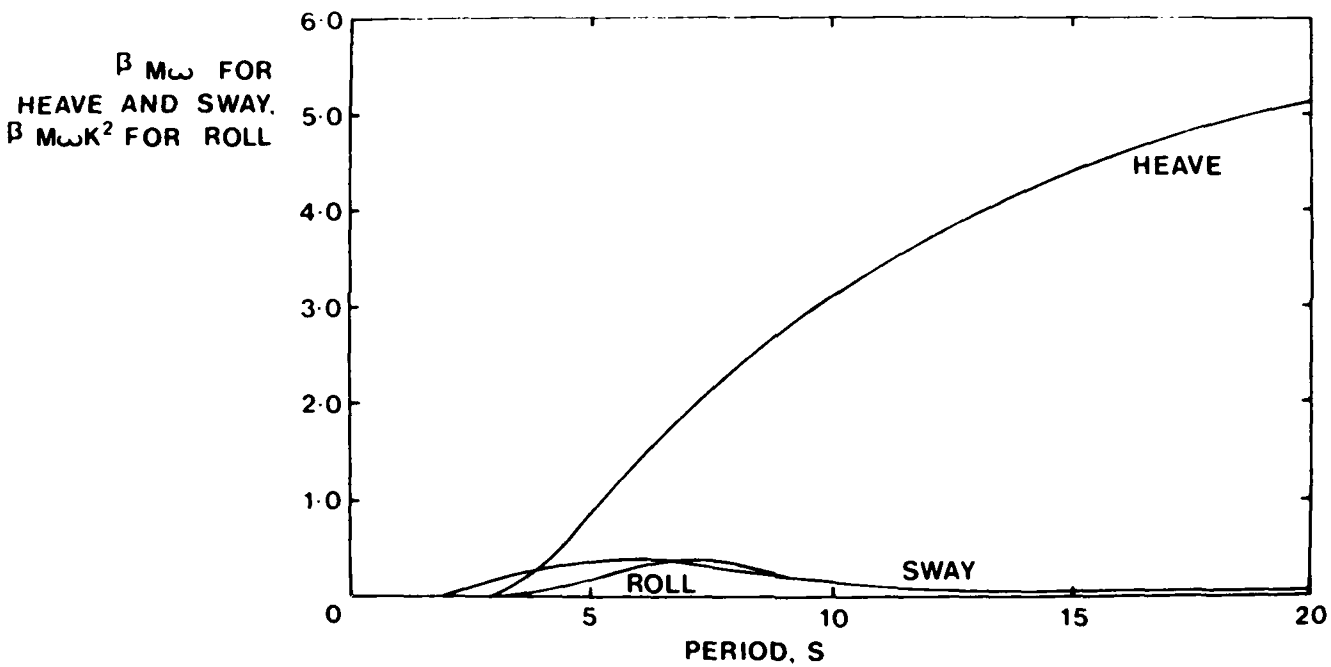


Fig. 2.3 The damping coefficients for a 20m by 3m ice floe in beam waves.

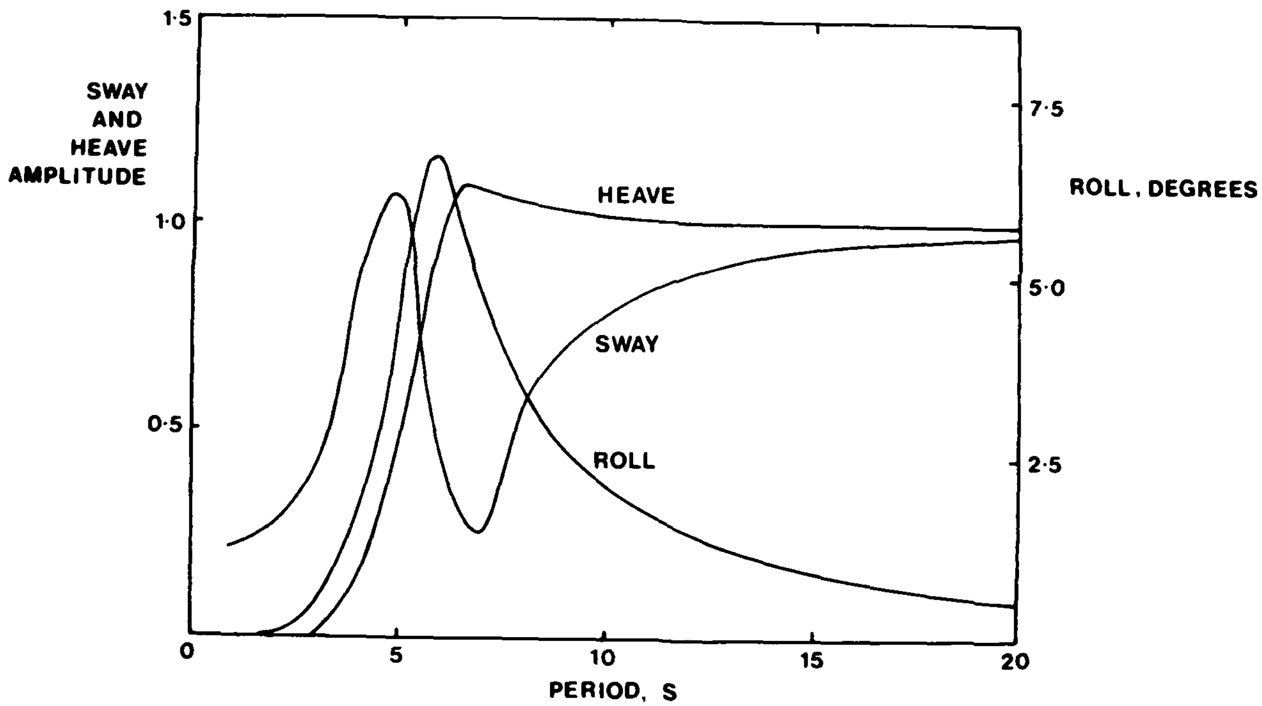


Fig. 2.4 The amplitude of the frequency response function (gain factor) for a 20m by 3m ice floe in 15s beam waves.

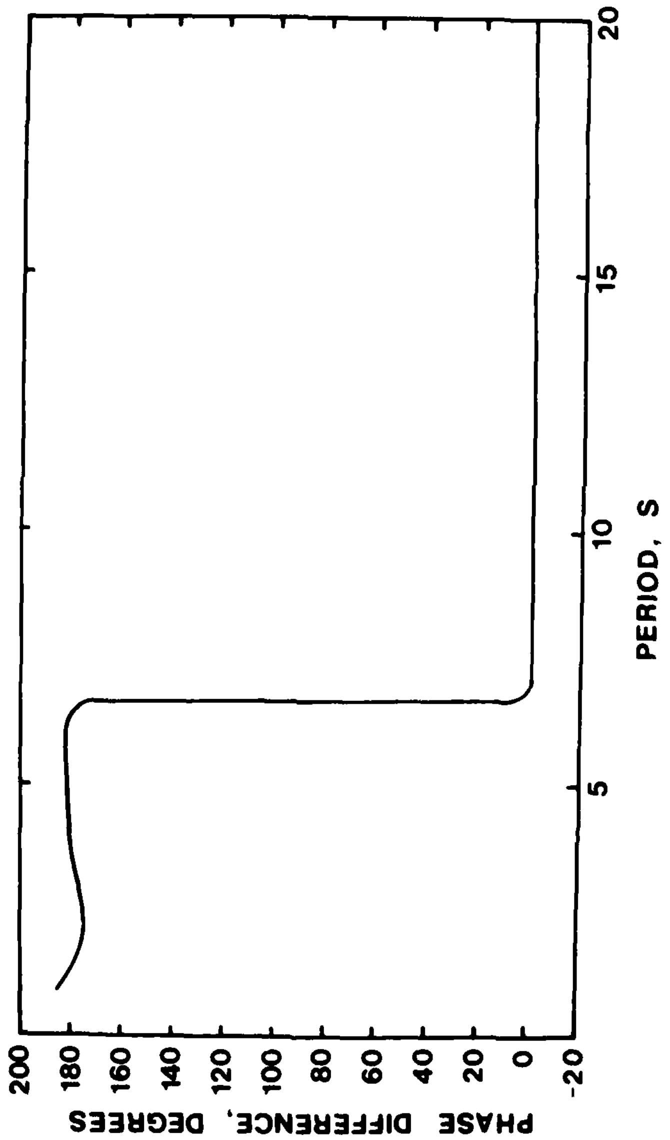


Fig. 2.5 Phase difference between roll and sway motions.

which appears to correspond to the minimum in the sway amplitude ratio curve, the phase difference rapidly changes through 180 degrees. Vugts found this phase change experimentally for various rectangular sections and in each case the phase altered rapidly near the sway minimum. At low periods therefore, our roll and sway are in anti-phase, whereas at longer periods they are in phase. Fig. 2.5 illustrates the complicated nature of the coupling which exists between roll and sway.

In fig. 2.6 we see perhaps the most important part of our numerical calculation as far as this report is concerned. Here we have plotted the various pressures beneath the ice floe due to the incoming waves. Each of the three graphs are plotted to different scale to emphasise the curvature of the pressure fields, so that care should be taken in interpreting the shape between graphs. The pressure fields are all plotted at time zero relative to a 15s wave with its crest at the mid-point of the floe. In the bottom figure we see the pressure contributed by each rigid body motion. The heave pressure field is symmetrical about the mid-point, whereas the sway and roll curves are anti-symmetrical as would be expected. The centre graph represents the underside pressure for an equivalent but restrained ice floe. In this case the curve is very neary symmetrical since the wave period is relatively long in comparison to the beam. For shorter periods the degree of asymmetry in the restrained pressures increases rapidly. The complete pressure field, being made up of the four component fields, reflects this asymmetry and is influenced by the anti-symmetric structure of sway and roll. For the 15s wave case shown, once again the asymmetry is small but it is significant as we will discuss in Section 3 where the rigid body assumption is relaxed and the pressure beneath is allowed to bend the floe.

In this section we have presented numerous graphs demonstrating the use of the recently implemented, hydrodynamic program for evaluating the motions of floating bodies in waves. We have not attempted a strict comparison with laboratory experiments for the technique is well-proven and the program well-tested. Rather, the approach has been to look at the values calculated by the program for the sort of ice floe we are likely to be working with, and to interpret those values qualitatively by means of laboratory experimental reports in the literature. The model revealed

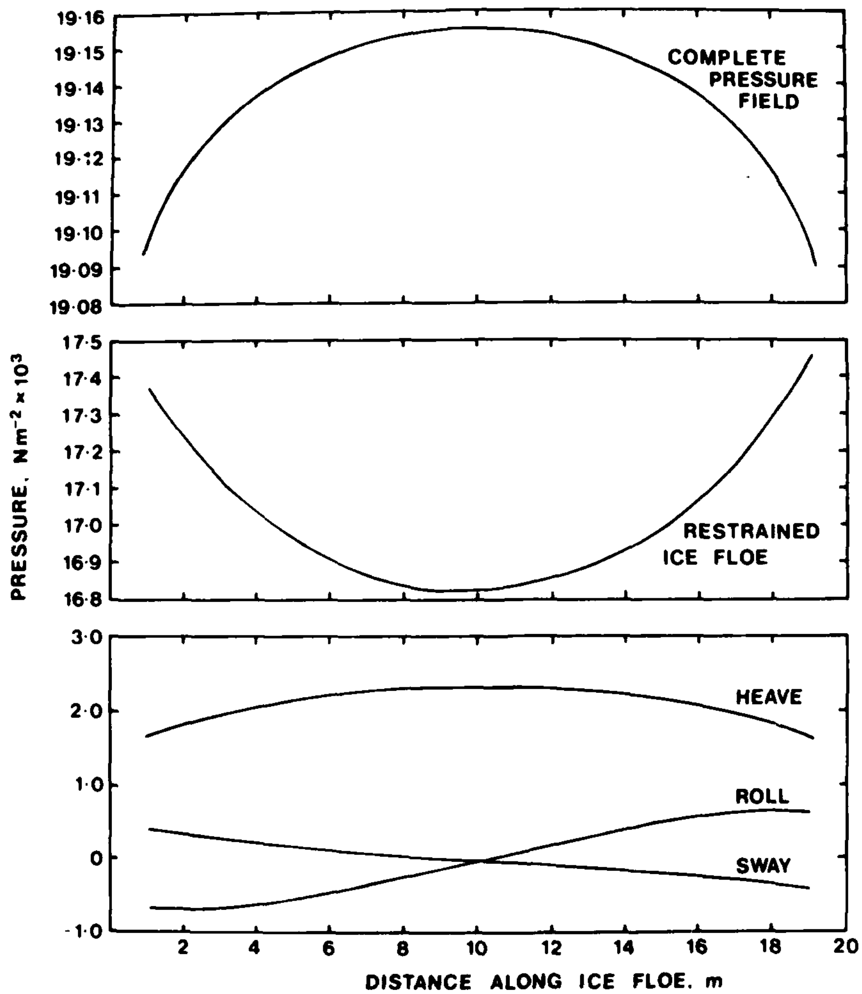


Fig. 2.6 The component and the complete pressure fields beneath a 20m by 3m ice floe due to 15s beam waves. Note the scale change in the pressure axis.

structure that could not have been predicted without an intensive series of scaled laboratory or field experiments, which would be prohibitably expensive. The existence of resonant peaks within the bandwidth of a typical open ocean spectrum is extremely relevant since this would considerably influence any experimental values measured aboard the ice floe. The anti-phase/in-phase relationship of sway and roll is also very interesting and is worth investigation under field conditions by means of tiltmeters and sway accelerometers.

3. FLEXURAL BEHAVIOUR OF ICE FLOES IN WAVES

3.1 Introduction

In the first part of this article we discussed and demonstrated a powerful method for evaluating the rigid body motions of an ice floe or iceberg subjected to incoming waves. The numerical method was able to cope with an arbitrarily shaped two-dimensional section of the body and compute five out of the six rigid body modes. The sixth mode, surge, has little effect on the pressure beneath the floe so can safely be neglected without undue loss of accuracy. In this section we aim to relax the fundamental assumption of rigidness and allow the floe to bend as the wave propagates beneath. The basis of our entire analysis is the hypothesis that to first order, the flexural behaviour and the rigid body behaviour may be decoupled. As far as the fluid is concerned, this is equivalent to saying that the particle motions beneath the wave profile do not feel the body bend. Thus, from a hydrodynamical point of view, our assumption is extremely reasonable since typical flexural displacements lead to extremely small floe curvatures which can have negligible influence on the fluid motions. Certainly the more important consideration is, what is the effect of decoupling our calculations as far as the ice floe's response is concerned? Essentially, the wave-ice interaction problem is a resonance problem (R.E.D. Bishop, personnal communication, 1981) so that feed back mechanisms between flexural and rigid body behaviour are not necessarily negligible particularly close to natural frequencies of oscillation or flexure. Naval architects have for many years tended to overlook these problems and have obtained quite reasonable results with relatively simple models (often assuming the Froude-Krylov hypothesis). We proceed with our more complete but decoupled solution with this in mind. Comparisons of our model with real data are regarded as the ultimate test of the decoupling procedure and will be carried out in a later paper. A preliminary comparison of data and theory is presented in section 4.

The rigid body program described earlier is not restricted in any way to simple geometrical cross-sections. As long as the ice beneath the waterline can be effectively modelled by a topologically simple curve, and hence by a collection of points on that curve, the program may be run and the modes and sub-ice pressures computed. It is clearly possible to then use these pressures to find the strain at the surface of a floating ice floe by means of a thin plate analysis (Timoshenko and Woinowsky-Krieger, 1959). The author began this work by adopting such a procedure. An elementary computer program was written based on the Duhamel integral method of Timoshenko et al (1974) and discussed more fully in Goodman et al (1980) with reference to ice islands and ice floes. The program is capable of computing the strain as experienced by a strain gauge located anywhere along the floes upper surface. Good agreement with observations for thin floes was obtained though the discrepancies between theoretical and measured strain became significantly worse as the thickness increased.

In many ways a thin plate theory is a step backwards since we have been forced to restrict a very general model to the oversimplified geometry of a thin plate. By doing this we have destroyed any capability of solving more interesting problems such as: how does a floe with a sill or an undercut bend on waves; what effect would a keel beneath the floe have on its total response; how would a cracked floe respond; etc. If we want to solve such problems, it is clear that no amount of analytical work will yield solutions for floes of such arbitrary shape and specification. We are forced to turn to numerical methods.

There are two popular numerical procedures by which this sort of problem may be solved. Both are extremely general and both could well be used to compute stresses and strains etc. for arbitrary shaped floes given some pressure loading. Both methods have well-defined advantages and disadvantages.

The first method historically is that of finite difference analysis, whereby the partial differential equations for the problem to be solved, and its boundary conditions, are expressed in their finite difference forms. A matrix equation may then be constructed and solved by means of some sort of iterative procedure such as successive over relaxation or an equivalent technique. The main drawback of this method is that it is difficult to generalise to arbitrary geometries since by its very nature, the matrix equation set up is specific to the geometry and the grid pattern chosen. This implies that any finite difference analysis tends to be very problem specific.

The second and more recent method was developed with the advent of large main-frame computers. In this technique, known as finite element analysis, the body is replaced by a "patchwork" continuum of smaller bodies with exactly the same physical properties. The choice of the word "patchwork" is deliberate for it emphasises the fact that each element has the same characteristics as the complete body. These elements are assumed to be interconnected along their boundaries at a discrete number of nodal points whose displacements prescribe the displacement of the element as a whole. By expressing the element's state of deformation (or strain) in terms of a uniquely defined displacement function, and by considering the appropriate nodal forces, boundary stresses and distributed loads, it is then possible to write down the so-called stiffness relations for each element and hence for the entire body. Clearly there are certain topological restraints which should be applied to the continuum since overlapping of elements or their separation to form holes or cavities cannot be permitted. Unfortunately, such restraints imply continuity of nodal displacements between elements which can only be satisfied in the limit of an infinitesimally fine discretization, so that a degree of approximation is inherent in the finite element technique. The matrix equations formed in this way may be solved to give the body's displacements, strains or stresses by one of the standard numerical procedures. For a more detailed description of finite element methods the reader is referred to Desai and Abel (1972) or Zienkiewicz (1972). It is this method that the author has chosen to model the ice floe in waves problem.

3.2 The Sub-Ice Pressure Field

Throughout this work we will assume that our ice floe is of simple rectangular cross-section. Typical east Greenland floe dimensions of 20m across by 3m thickness are chosen as before though of course the simplicity of the geometry and the particular dimensions chosen are unimportant. With this model, a rigid body calculation as described in part 1 of the report may be carried out and the pressure field computed beneath the floating ice floe for a variety of incident wave periods. For a 20m floe, 16 nodal points will produce the pressure field as shown in fig. 3.1. The ice floe can then be allowed to bend to this pressure field.

From the point of view of surface strain, the edge pressures will have little effect on an instrument located near to the centre of the floe. The edge pressures are therefore neglected for the present analysis. Indeed, these pressures will very nearly cancel one another out as far as the rigid body part of the calculation is concerned (in a symmeterised analysis they would be equal and opposite). The pressures beneath the floe are defined at isolated positions along its underside, but none-the-less they must represent a smooth and continuous upward pressure distribution. This distribution will be slightly asymmetric in a complete analysis since one would expect the wave to be effected by the ice floe. It would be quite feasible to use the sampled pressure field to solve the bending problem. However,such a grid spacing would be extremely coarse and would no doubt produce unacceptable accuracy when the computation was carried out. For this reason, the author has chosen to use the sampled pressure to regenerate the "original" distribution by means of interpolation. The method employed for this reconstruction is a well-tested numerical technique which creates an interpolate curve by patching together a series of cubics. The interpolate is continuous and has continuous first derivatives. By this method, the upward loading due to the static wave profile may be found at any point along the underside of the ice floe. In principal the loading may then be applied to a finite element model and the resulting displacements and principal stresses found.

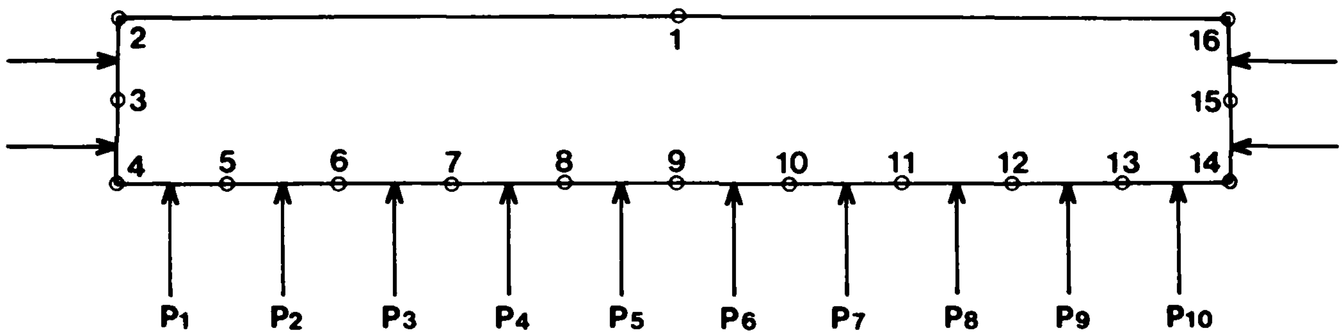


Fig. 3.1 The discretized pressure field beneath the ice floe showing the chosen nodal pattern.

3.3 Preliminary Discussion of the Finite Element Model

In practice it is somewhat more complicated to solve the equivalent finite element problem. Three important questions should be asked: a) if we have reduced the ice floe to an equivalent finite element representation, do we have to redistribute our loading in any way so as to effectively model the upward pressure with minimal error; b) it is clear that the pressure beneath the ice floe will tend to move the ice floe upward as well as bend it, is such a displacement permissible and if not, how do we cope with it; c) does the fact that the ice is floating in water significantly effect the stress/strain calculations? Each question will be answered in turn.

a) Distributed loading: Despite our having interpolated the pressure loading so that we are free to compute the upward force at any point beneath the floe, it is still effectively a sampled distribution. This implies that once we have settled on our finite element spacing, we still have at best a trapezoidal-type loading pattern (fig. 3.2). One possibility would then be to average the loads in some way along the under-ice element boundaries. This method is inaccurate however and a better way is to compute the distributed load which does the same work on the finite element grid as the pressure distribution would do on the real ice floe. For trapezoidal loading, the calculation involves the combination of uniform and linearly varying pressure. Suppose we have a set of elements as shown in fig. 3.3, then the equivalent distributed loading for a trapezoidally changing pressure is as shown where the composite loads are applied at mid-side nodes. The method for computing this algorithm is not dissimilar to that of Simpson's rule of numerical integration. With the above loading pattern, applied both at the corners of elements and at their mid-sides, the work done by the load on the mesh is equivalent to that done by the original pressures on the ice floe. By this means, the maximum accuracy possible will be retained.

b) Rigid Body Displacement: Initially, the rigid body effect produced by the upward pressure distribution was disregarded, and the stresses and strains computed at the maximum machine precision possible. This method is unsatisfactory for two reasons: first that for any graphical output,

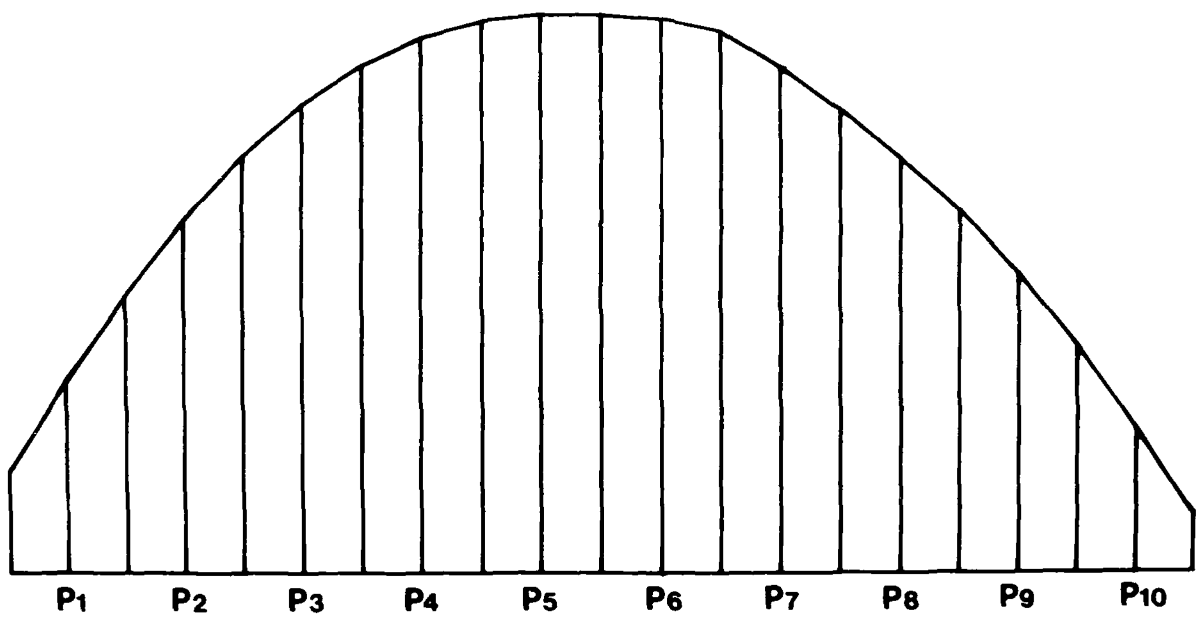


Fig. 3.2 Trapezoidal loading pattern for pressure beneath an ice floe. The pressure curve is approximated by straight line segments.

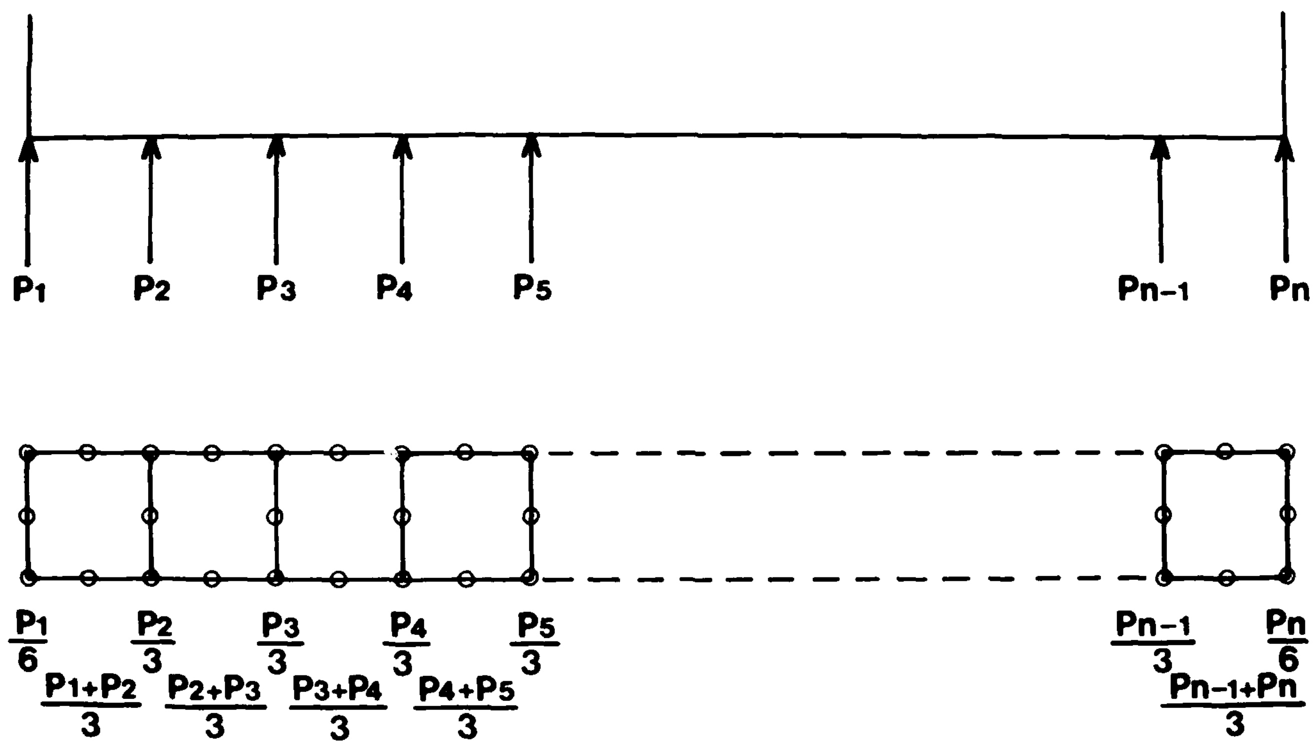


Fig. 3.3 Equivalent nodal loading pattern for the finite element model.

the bending is buried within the much larger rigid body translations; and secondly that computing time substantially increases when maximum precision is used throughout. It was soon clear that some alternative procedure was necessary. R.D. Henshell (personnel communication, 1981) suggested that either the rigid body part of the loading should be removed so as to produce an integrated load which did no work on the ice floe, or an enhanced gravity field should be applied so that the total upward pressure was opposed exactly by the downward body forces. The first method is by far the simpler to implement routinely so that the present author has adopted that method. The original and the transformed pressure distributions are shown schematically in fig. 3.4 where in the second diagram the shaded area must sum to zero. When this procedure is implemented, increased precision computation can be limited to the displacement calculations alone, and the stresses and strains may be found using less computer time and machine storage.

c) From a common sense viewpoint, the fact that the ice is afloat must be significant since without the presence of the underlying water, an unrestrained ice floe would simply move skyward an indefinite amount under upward loading. One solution might be to restrain the body at some node within the finite element mesh. Such a procedure would yield reasonable bending displacements but introduces unrealistic stress concentrations near the constrained node and is therefore unsatisfactory. A better procedure would be be to consider exactly what happens when a floating body is forced upwards by a small distance. Suppose that our ice floe is raised by an amount δ , then an opposing pressure of magnitude $\rho_w g \delta$ will act so as to restrict translation. This implies that the water beneath the floating body is behaving as a spring with modulus equal to $\rho_w g$. We say that the ice is behaving as though bonded to an elastic foundation of modulus $\rho_w g$. Returning to the finite element model, therefore, we represent the flotation part of the problem by connecting a series of grounded springs to the elements on the underside of the ice floe. The equivalent distribution of springs (so that the spring loading does the same work on the finite elements as would the real buoyancy forces on the floe) is computed by assuming a uniform downward loading of the mesh.

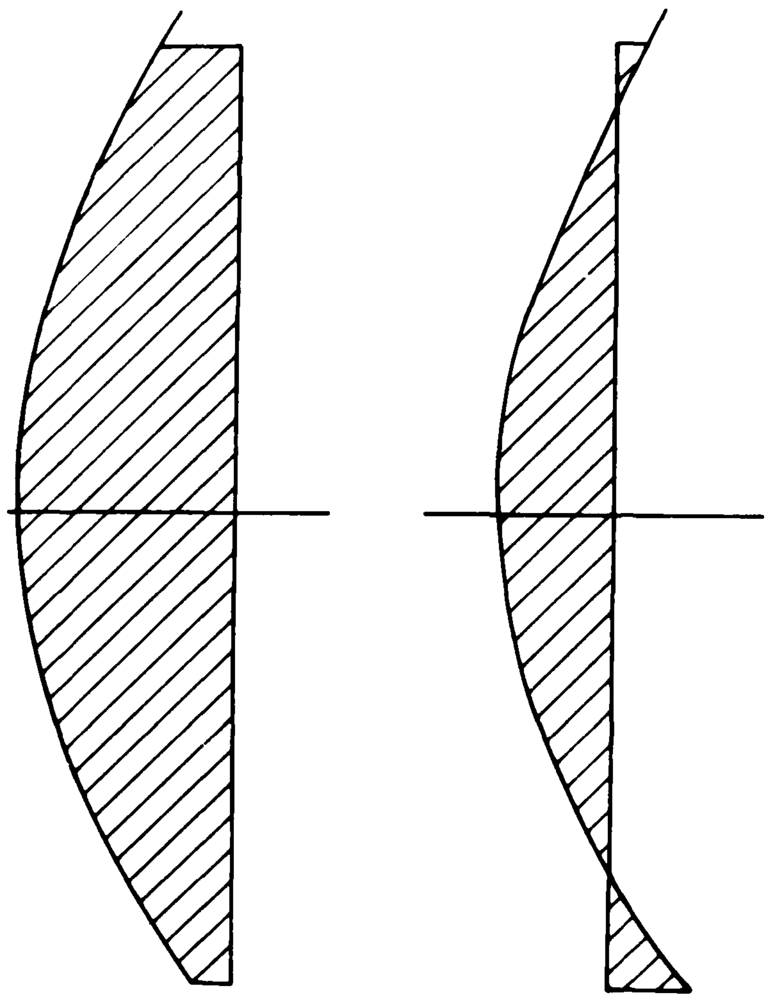


Fig. 3.4 Schematic sketch showing removal of rigid body part of pressure loading.

We have discussed three basic questions which are encountered when the finite element method is applied to ice-wave interactions. We may now proceed with a more detailed discussion of the method employed.

3.4 Application of the Method

By its very nature the rigid body calculation discussed in the previous section is a two-dimensional strip theory. That is to say that if three-dimensional results are required, the body is assumed to be made up of two-dimensional elemental strips or cross-sections which may be totally decoupled from one another. The program may then be run for each cross-section and the results integrated so as to simulate a three-dimensional floating body. The strip theory has been fully tested over the years and has proved reliable in computing the rigid body behaviour of a variety of underwater forms. Indeed, there is little justification in developing a fully three-dimensional theory since the theoretical and the measured results show excellent agreement (C.M. Lee, personnal communication, 1980).

When one allows the body to bend under the wave loading however, the effectiveness of a two-dimensional model is not nearly so well-defined. The author knows of no equivalent technique which has been compared with experimental data in any way, so that although it is tempting to treat the bending as a strip theory, such a step must be regarded as an untested hypothesis. It should also be noted that whereas the reduction of the full three-dimensional rigid body analysis to a series of two-dimensional strips is conceptually easy to grasp, particularly for beam waves, this is not the case when a body is allowed to bend. In general, stress and strain are tensorial quantities and when simplification is made into a planar or cross-sectional analysis, care must be taken in both defining the problem and in interpreting the results. This is particularly true when icebergs are considered since all three dimensions are of comparible magnitude. It is important to appreciate that the behavior of a two-dimensional section is not the same as that of an equivalent plate of arbitrary thickness. The first case is a problem of plane strain whereby any displacement into the third dimension is assumed to be zero, and the second case would be solved as a plane stress problem so that the components of stress into the plate thickness would vanish. Our cross-sectional strip of ice floe should be solved in plane strain which fortunately leads to a very simple relationship between the principal stresses σ_1, σ_2 and the principal strains ε_1, ε_2 material (Jaeger, 1956), viz.

$$\begin{aligned} \sigma_1 &= (\lambda+2G)\varepsilon_1 + \lambda\varepsilon_2, \\ \sigma_2 &= (\lambda+2G)\varepsilon_2 + \lambda\varepsilon_1, \end{aligned} \tag{3.1}$$

where λ and G are Lame's parameters, and the convention that the subscripts 1 and 2 imply greater and lesser principal components will be used. For icebergs however, the plane strain assumption could lead to serious errors if the length, breadth and thickness are similar, and strictly a completely three-dimensional approach should be adopted. With the limitation of a two-dimensional rigid body strip theory, this might be carried out in three stages: first the iceberg is considered as a series of cross-sectional shapes and the pressure loading is calculated for each section; then an interpolate surface is fitted to the pressures in a similar way to the pressure curve found in our two-dimensional study; finally the pressure surface is used as areal loading in a three-dimensional finite element model. The author does not propose to demonstrate the use of this technique in this report since we are primarily concerned with ice floes.

The numerical scheme for carrying out the finite element analysis was developed by PAFEC Ltd. in the U.K. It consists of a suite of user orientated Fortran programs capable of dealing with a variety of engineering-type problems and in principle may be used with little difficulty once the fundamentals of finite element modelling are understood. It is fair to say that we have used finite element modelling in rather an unorthodox way; we have applied a very powerful engineering technique to a complex geophysical situation. All the difficulties encountered have been conceptual rather than fundamental.

A suitable grid pattern for our 20m by 3m ice floe was set so as to take advantage of the geometric simplicity of the original rigid body nodal pattern. Twenty elements were chosen to represent the floe lengthwise and ten elements through its thickness. Tests with finer grids showed little improvement in accuracy. The finite element mesh is shown in fig. 3.5.

A preliminary numerical simulation was carried out by reducing the dynamic loading to a static profile defined at the time of maximum bending. This is by no means a trivial step especially for short period waves, since there exists a phase lag between the original wave passing beneath the floe and the resulting pressure distribution. When comparing waves of different periods, therefore, care must be taken to ensure that the computed strains have been calculated relative to equivalent parts of the loading. If the pressure distribution were a simple sinusoid it would be a simple matter to apply the necessary phase shift. For complicated pressure fields however, we require some criterion by which to decide how much the phase should be altered so as to standardise the flexure. The method adopted uses a finite difference iterative procedure to compute maximum numerical curvature at the centre of the floes upper surface as the wave moves relative to the floe. The wave is then shifted by the phase necessary to attain this curvature. The phase shift for long period waves is negligible but as the period decreases so the phase change becomes more significant. After this phase translation has taken place, the maximum stresses and strains would be found near the centre of the top of the ice floe.

The static analysis has been carried out for several wave periods in the range 5s to 25s passing beneath our 20m by 3m floe. For a 15s wave, the loading is close to being symmetric but as can be seen in fig. 3.6, the asymmetry of the under-ice pressures is sufficient to slightly tip the ice floe on the wave. Fig. 3.6 shows the resultant displacement of every point in the finite element grid representing the floe. Clearly there is significant bending and, as one might expect, the maximum bending displacement is to be found at the centre of the floe's upper surface. Returning to the apparent rigid body rotation in the figure, we see that the angle of inclination is less than one second of arc which seems quite reasonable for the initial asymmetric pressure pattern.

Having computed the displacements, we are now in a position to find the stress or strain field within the ice floe. Perhaps the simplest way of visualising stresses is by means of a stress vector plot. In fig. 3.7 the

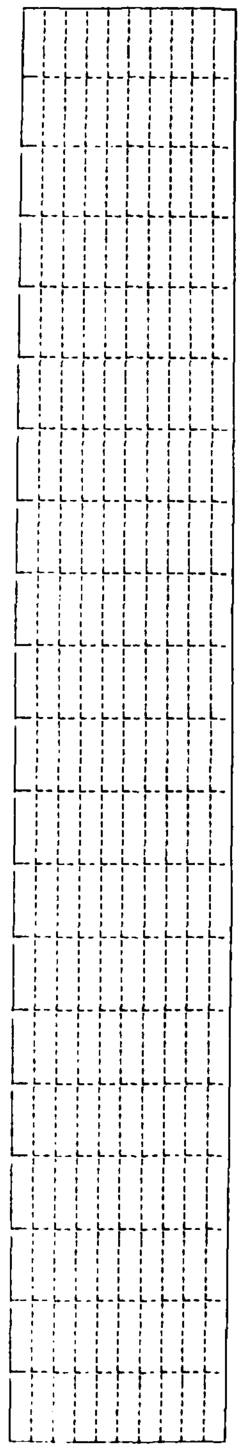


Fig. 3.5 The chosen finite element mesh for the 20m by 3m floe.

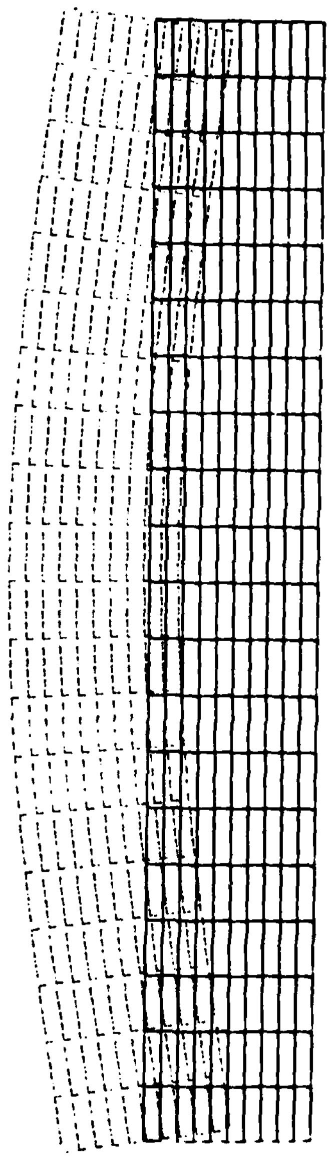


Fig. 3.6 The flexural displacement due to a 15s wave. Note the slight rigid body translation.

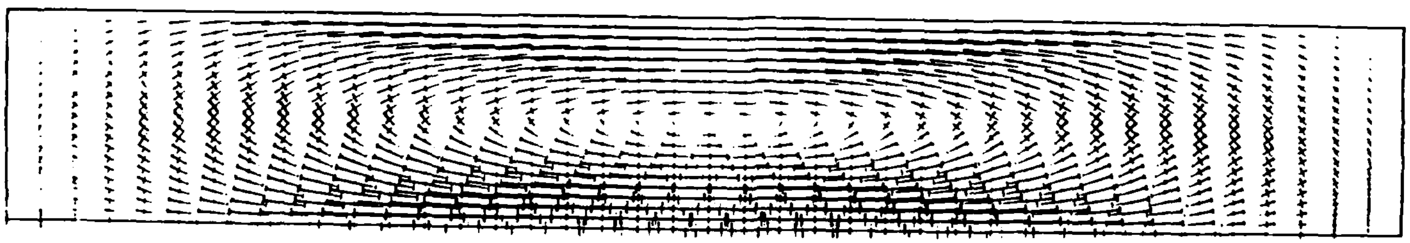


Fig. 3.7 Stress vector plot for ice floe. The length of each vector is proportional to the stress at that point and the inclination is equal to the orientation of the axes of principal stress. The scale is 219.8 Nm^{-2} per cm. Vectors with end bars represent compression, vectors without end bars represent tension.

vectors shown have a length which is proportional to their magnitude and are plotted with the correct angular translation. Both principal stresses are shown with the convention that compressive stresses are drawn with end bars whereas tensile stresses are not. The principal stresses form the pattern that one would expect for a simple body of this sort. We see extensional stresses above the neutral axis and compressional stresses beneath. In a fully dynamic analysis this situation would reverse mid-cycle.

An alternative means of displaying the stress pattern is by means of a contouring routine which draws lines of constant stress on the two-dimensional cross-section (fig. 3.8). Here we see that so long as we do not venture too close to the edges of the ice floe, the isopleths are parallel to the upper and lower surfaces. Such a result would be expected in a thin plate theory. The contouring routine shows clearly that the maximum tensile stress is to be found at the central position on the floe's upper surface, and the maximum compressive stress located directly beneath that on the underside.

Bearing in mind that ultimately we wish to test our model with the Sea Ice Group's strainmeters, we have converted the principal stresses to principal strains by means of equations (3.1). This has been carried out for all nodes along the floe's upper surface where the smaller principal strain is several orders of magnitude smaller than the larger. The directions of the axes of principal strain are such that the larger strain is horizontal. Fig. 3.9 shows this strain along the ice floe. From this graph we may say that the maximum strain experienced by a 20m by 3m floe due to a 15s wave of 1m amplitude is 5.5×10^{-8} and that this value occurs near the floe's centre. As an aside we may use this value to determine whether or not such a wave could fracture the floe and if not, what amplitude would be necessary to break it. With an empirical fracture strain of 3.0×10^{-5} (Goodman et al, 1980) we see that fracture is unlikely to occur.

The change in strain through the floe is shown in fig. 3.10. As one would expect for an isotropic material, the strain vanishes mid-way through (i.e. on the neutral axis), and the largest absolute principal

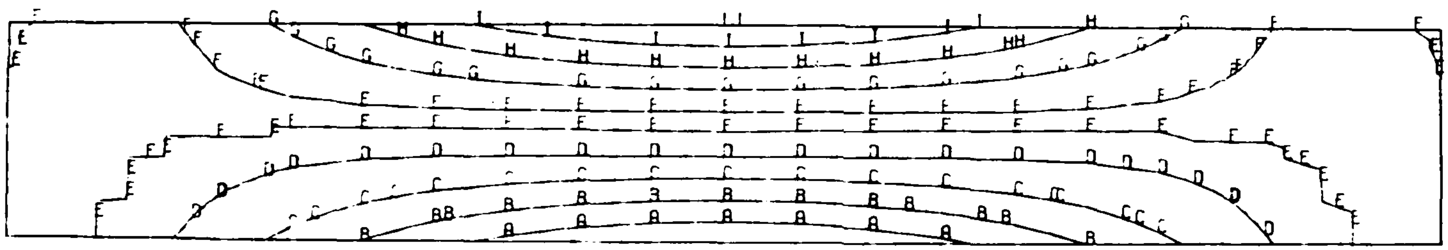


Fig. 3.8 Contours of largest principal stress due to wave loading.
Key: A=-287.0; B=-215.0; C=-143.0; D=-71.4; E=0.63; F=72.6; G=144.0; H=216.0; I=288.0; J=360.0. All units are Nm^{-2}.

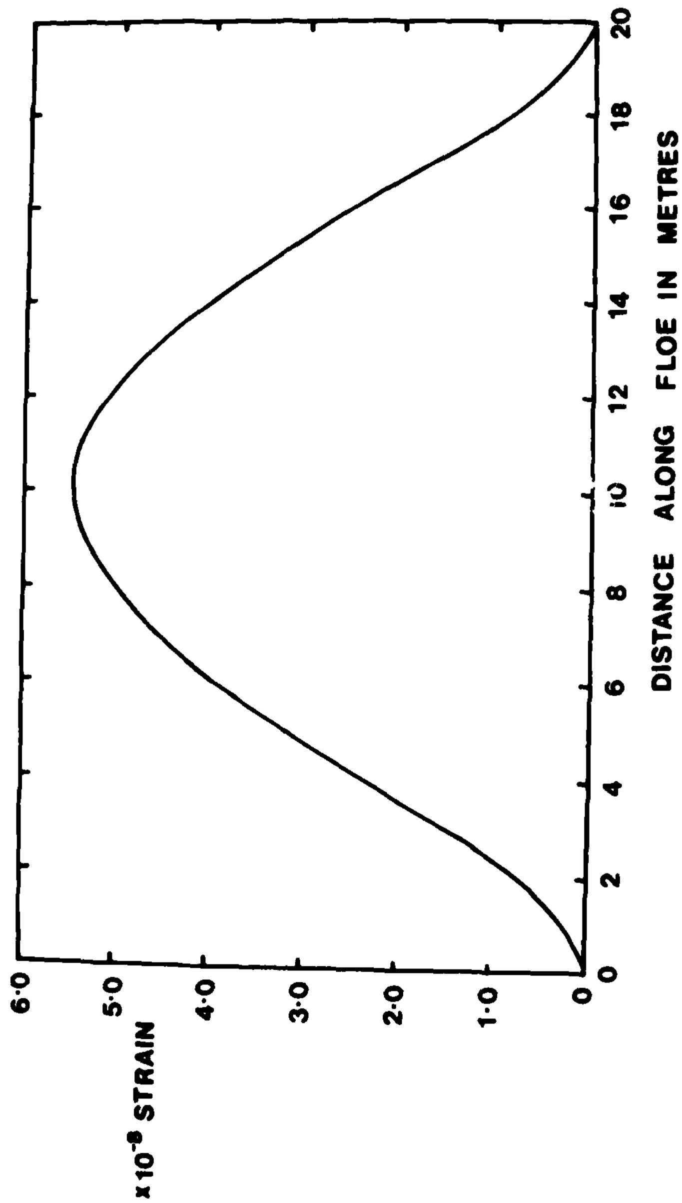


Fig. 3.9 The surface strain along the ice floe due to a 15s wave beneath.

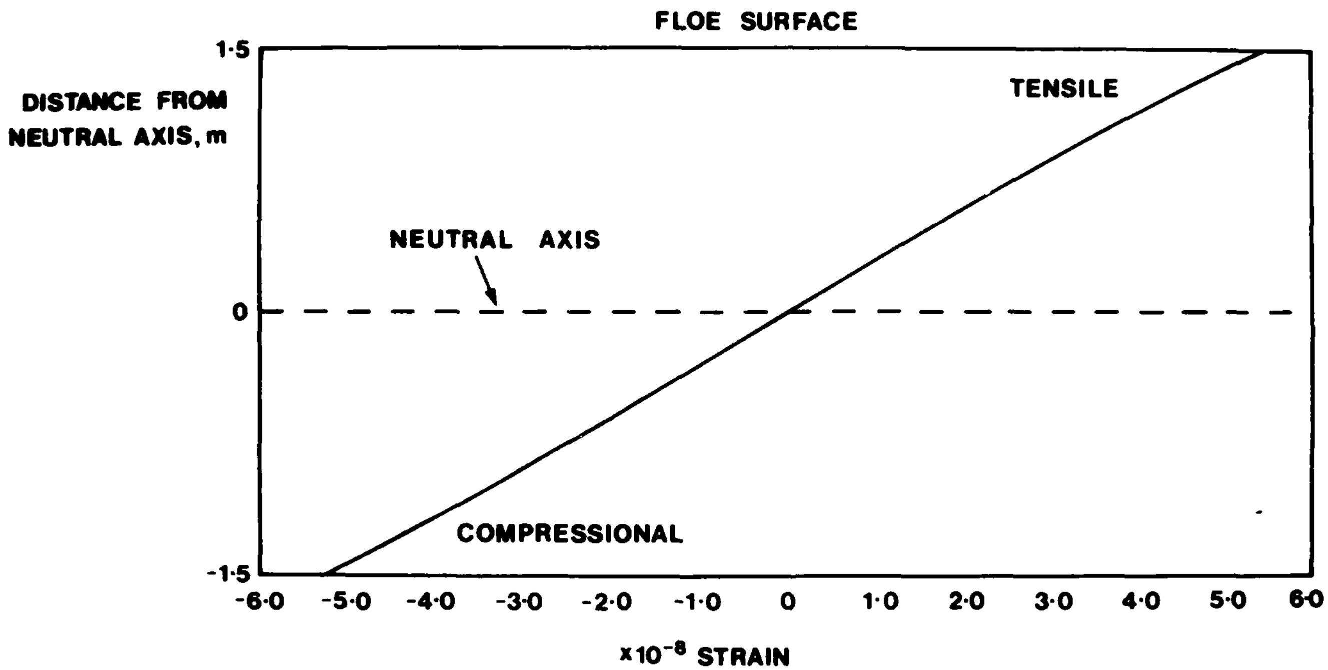


Fig. 3.10 Regions of tensile and compressive strain for a 20m by 3m floe of rectangular cross-section under wave loading.

strain at the surface is equal in magnitude but opposite in action to the equivalent under-ice strain.

A novel way to view the strain field is shown in fig. 3.11 where the tensile and compressive principal strains have been plotted as surfaces. The neutral axis is clearly marked on each three-dimensional representation. The plots should be interpreted with reference to fig. 3.7 where the orientation of the principal axes may be seen in the angular variation of the stress vectors (for a isotropic material in plane strain, the principal axes of stress and strain are aligned).

The final figure of this subsection, fig. 3.12, shows how an ice floe of twenty metre beam and three metre thickness will behave as ocean waves of various periods pass beneath. The curve plotted is normalised relative to an incident wave amplitude of 1m so that this should not be regarded as the floe's response to a spectrum of waves, but rather as the magnitiude of the frequency response function or gain factor (Bendat and Piersol, 1971) of time series analysis. The curve represents surface strain as measured by an imaginary instrument located at the centre of the ice floe. As one might expect, the strain at different wave periods is by no means constant. For very long waves the radius of curvature of the wave is large so that the floe bends only by a small amount leading to small surface strains. For short waves, the radius of curvature is small and in principle one might expect very large strains. This is not the case however, since both the magnitude and the gradient of the pressure along the bottom of the floe decrease rapidly as the period becomes very short. There is an optimum period which will produce maximum strain and therefore maximum likelihood of fracture. For the present floe this period is about 4s which is close to the period at which resonance occurs for heave, sway and roll. This does not necessarily mean that if energy of that period is available in the spectrum it will fracture the ice floe; rather we are saying that if the spectrum were such that all frequencies were represented by equal amounts of energy and that energy was sufficient to break the floe, then waves of period 4s would be the most likely to do so.

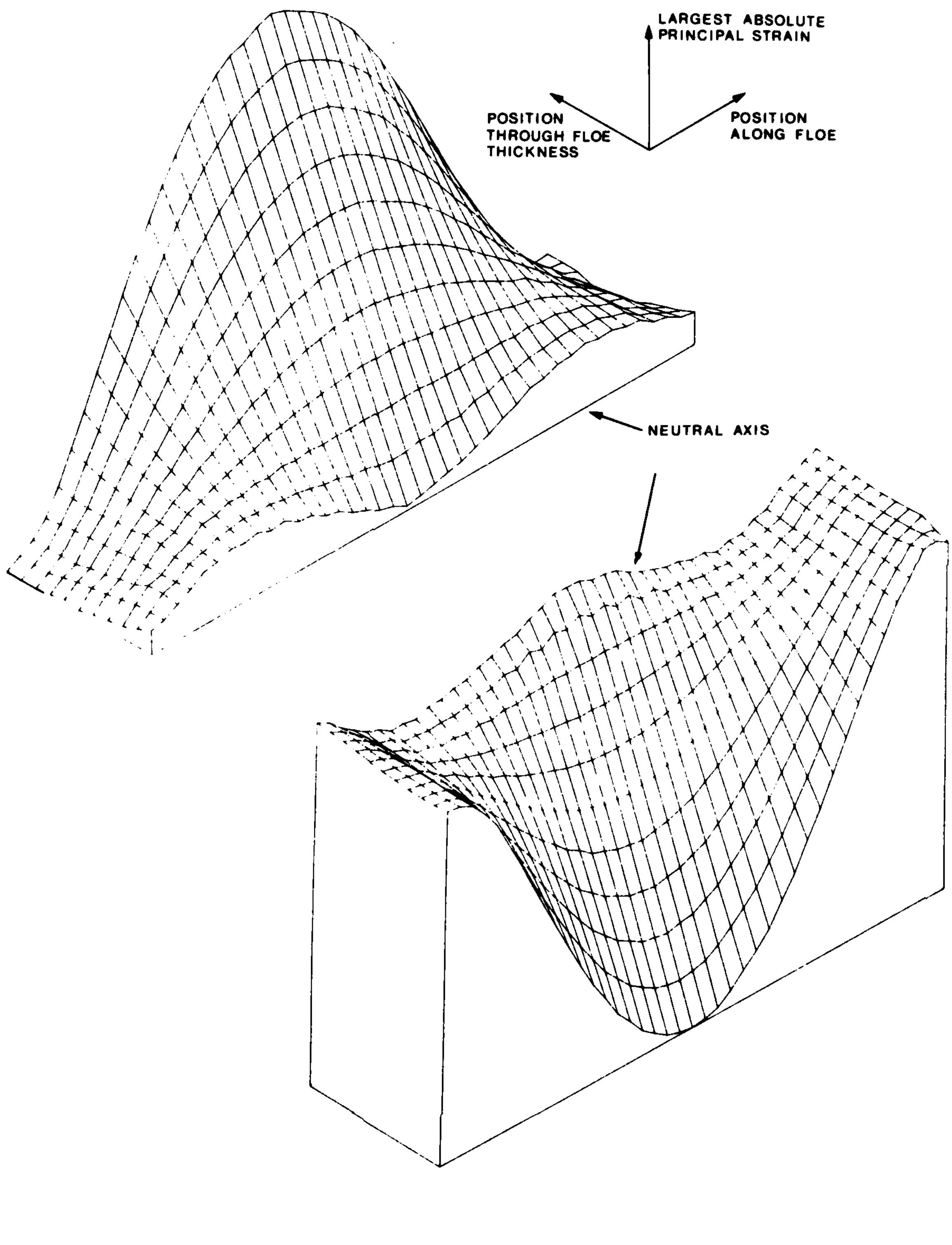


Fig. 3.11 Three-dimensional schematic showing the largest absolute principal strain through the ice floe.

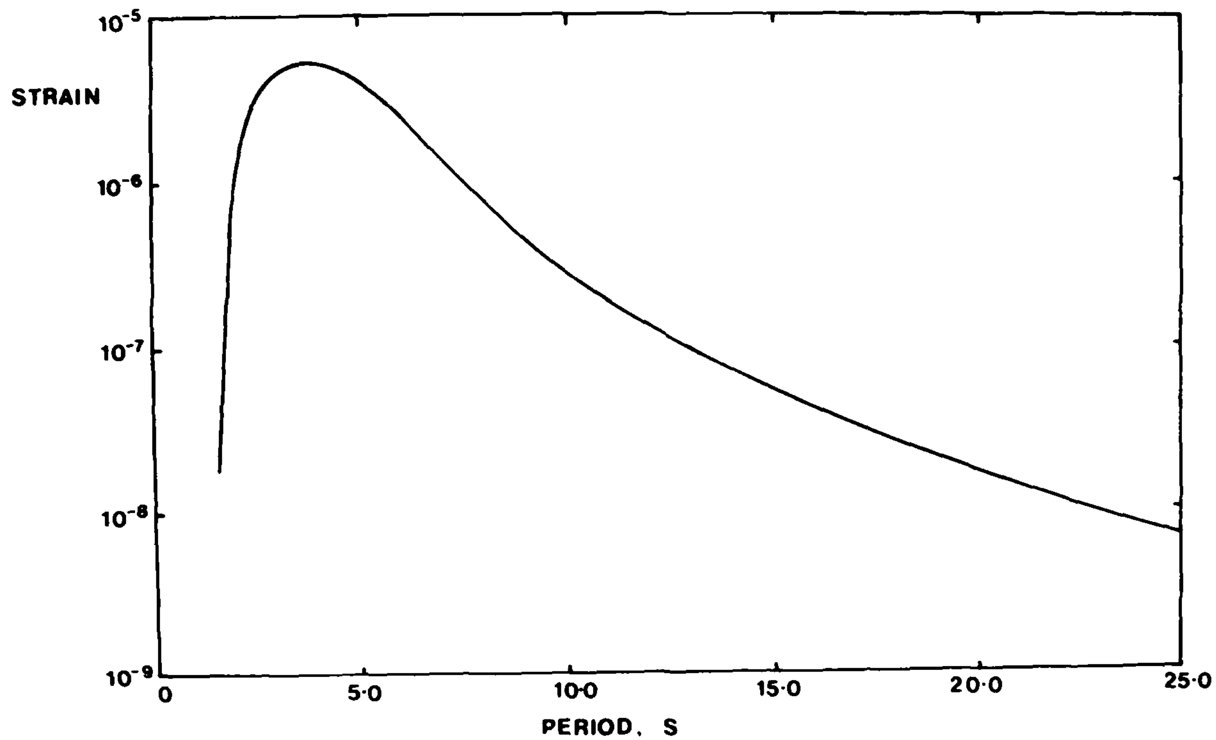


Fig. 3.12 Gain factor for maximum surface strain as a function of period.

3.5 Dynamic Analysis

The limitations of a static theory are not nearly so bad as one might first suppose. So long as the pressure loading does not tune to the floe's natural frequencies, the computed displacements and strains are reasonable and are believed to accurately model the flexure. However, static analysis tells us nothing about those natural frequencies so that resonance in flexure, which could foreseeably lead to the floe's ultimate destruction, is glossed over. Ideally, one would like to be able to compute the cyclic strains induced as the wave propagated beneath, look at those strains with passing time, possibly allowing the ice to deteriorate with time by some sort of fatigue mechanism, and then at the end of the simulation say something about the floe's survival. The problem is of course that such an analysis is prohibitably expensive and even if run, the results would be questionable due to the uncertainty of the physical properties of sea ice. For these reasons the author has not attempted such a mammoth task as implementing the necessary programs. The resonance problem still remains however, and it is important to be able to discuss the natural frequencies if one is to fully interpret the static analysis.

We first pose the question: what is the fundamental frequency of flexural oscillation and what are the frequencies of the first few harmonics for an 20m by 3m ice floe floating in calm water? We will refer to these frequencies as modes of oscillation. With no restraints applied there will exist three rigid body modes (for a two-dimensional section) which correspond to heave, sway and roll. In the present discussion we suppose these modes to be irrelevant and will consider the first mode of interest to be the primary flexural mode. We shall also limit our study to bending or flexural modes since more esoteric oscillations, such as tensional/compressional vibrations which might be induced by an explosion within the ice, are unlikely to be excited by ocean waves.

The determination of the natural frequencies of a body can be simplified so that it is equivalent mathematically to a symmetric real eigenvalue problem. For a finite element mesh which reasonably represents the body, one would expect a large number of nodes to be present. Each node is permitted, for a two-dimensional section, to move in two orthogonal directions known as degrees of freedom. It is clear that even though the problem can be reduced to a symmetric eigenvalue problem, we are likely to need a very large number of degrees of freedom to produce accurate results so that our matrices are huge. This would lead to unacceptably long CPU times and worse, to rounding errors.

Fortunately, many of the degrees of freedom in a large eigenvalue problem have little effect on the eigenvalues themselves. We may therefore simplify our analysis further by reducing the problem to finding the natural frequencies of an equivalent structure with less, but carefully selected, degrees of freedom. The selection of the remaining "master freedoms" is by no means simple and will not be discussed here; PAFEC includes a facility to do this.

A modal analysis of our ice floe was carried out so as to compute its natural frequencies. The first few modes are listed below:

MODE	FREQUENCY, HZ
1	19.13270 $\pm$ 0.00013
2	47.28252 $\pm$ 0.00002
3	82.75792 $\pm$ 0.00001

These modes correspond to fig. 3.13a,b and c respectively. The error bounds are approximate but have all been calculated so as to include the numerical error. It must be emphasised that these errors are an estimate of numerical accuracy. They do not in any way bound the absolute error caused by the approximations involved in using a finite element representation of the "real" ice floe.

Since the natural frequencies of any structure form an orthogonal set, any general distortion of the body may be regarded as the infinite sum of the modal distortions (for a system with an infinite number of degrees of freedom). In any real analysis this summation may be discontinued after the first few modes since the series converges rapidly. Our ice floe is no execption and we will retain only the first few frequencies in subsequent

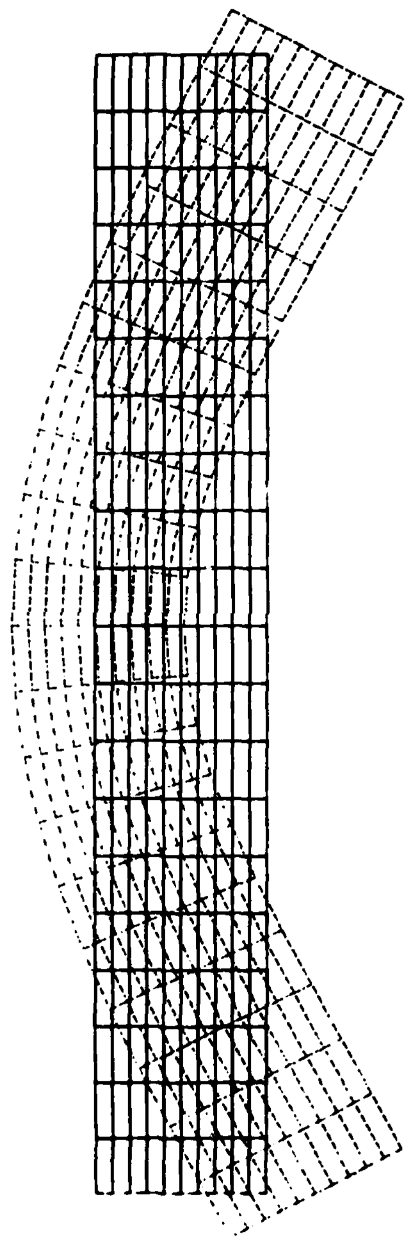


Fig. 3.13a First natural frequency of flexure for 20m by 3m floe.

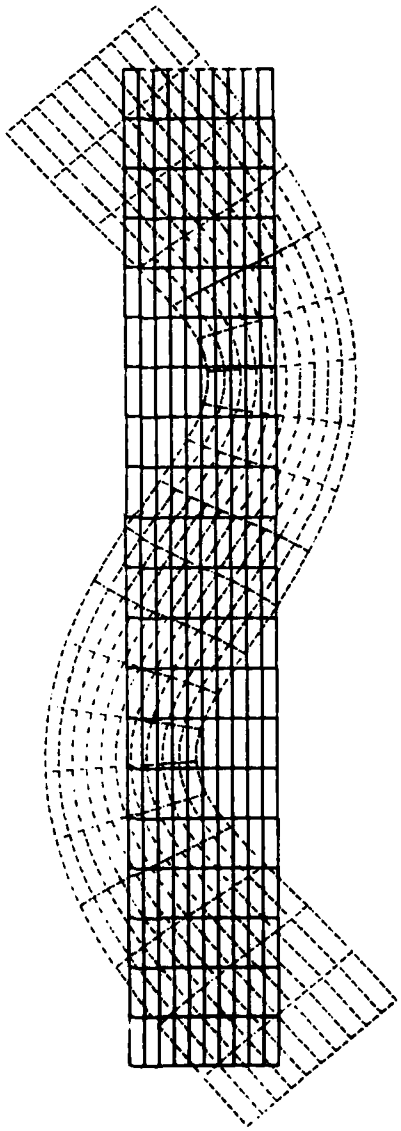


Fig. 3.13b Second natural frequency of flexure for 20m by 3m floe.

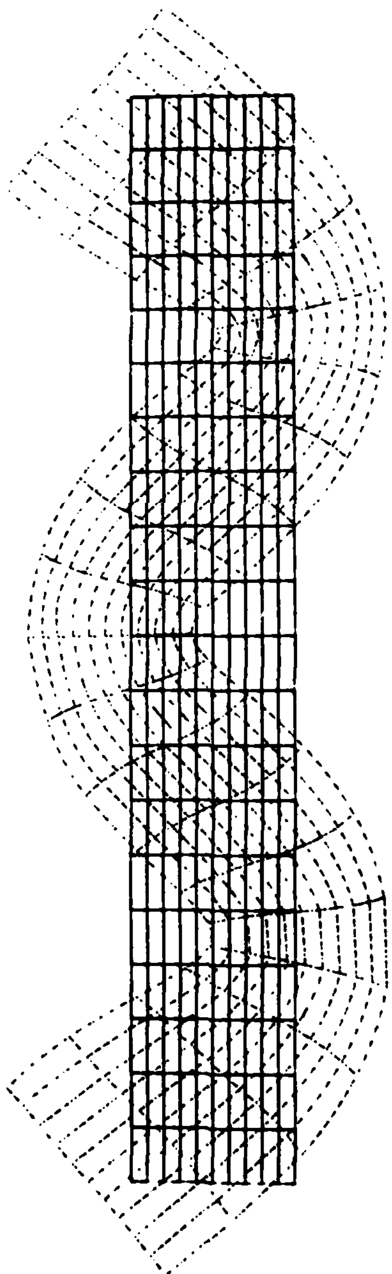


Fig. 3.13c Third natural frequency of flexure for 20m by 3m floe.

calculations [1] . We are now in a position to reconsider the wave loading and to determine the dynamic response due to waves passing beneath. If any of the wave periods encountered were close to a mode, resonance would occur, but in any realistic situation frictional forces within the material would damp the resonance sufficiently to prevent infinitely large strains. However, the resonant stresses produced might well be large enough to permantently weaken the ice floes and to ultimately cause it to break up. A secondary effect of the damping is to shift the body's natural freqencies so that the incident waves could foreseeably tune to modes at lower frequencies. This shift is second order however, and for small damping ratios is negligible. The most convenient way to impose damping on the system of equations is to allow the material's physical properties to become complex quantities. For the 20m by 3m ice floe, the modes occur at frequencies which are well outside the significant energy regions of typical open ocean spectra, so that resonance is unlikely. For this reason damping is really only necessary for calculations where broad bandwidth spectral observations are applied. In this case there is a possibility that a minute amount of high frequency energy present in a spectrum might lead to an unrealistic resonance.

As the size of the floe increases, so the natural frequencies decrease though the modal shapes remain the same. For an iceberg, there is a distinct possibility that the frequencies will shift sufficiently to be within the bandwidth of the wave energy of the sea. The tanker Pine Ridge broke in two in the Western Atlantic during December 1960 due to resonant stresses set up during a storm, so icebergs of similar and larger dimensions might well break-up by the same mechanism. When the natural frequencies of flexure are computed for a typical Antarctic iceberg it is found that the first flexural mode could easily be encouraged to resonate by the available ocean wave energy. Ms. Monica Kristensen, a research student at SPRI under the author's supervision, is currently considering this mechanism in the interpretation of iceberg data obtained during a recent Antarctic cruise aboard HMS Endurance. Her work will be published

---

[1] The number of master degrees of freedom chosen for the dynamic solution is finite so that we have already approximated the sum to some extent.

in due course. In conclusion then, the author agrees with the original suggestions by Goodman et al (1980) that <u>resonant</u> stressing due to waves is a significant factor if not the predominant mechanism in the ultimate destruction of icebergs.

Returning to our discussion of sea ice we emphasise that resonant stressing is unlikely to occur with any real ocean loading since the dimensions of typical ice floes are too small. In fig. 3.14 the vertical displacement and phase response of some arbitrary point within a 20m by 3m ice floe are plotted alongside the equivalent static response at various wave loading frequencies. The curves plotted have been normalised so that they are with respect to incident waves of unit amplitude. The <u>small</u> difference between the static and dynamic results is most probably due to rounding error, and contamination by the rigid body modes in the dynamic solution. These errors will increase as the wave period decreases. Even so, the reader will see that the curves agree well and that no resonance is occurring within the bandwidth of energy considered. This confirms that the destruction of ice floes by resonant stressing is unrealistic, and further demonstrates that our static model is adequate for stress or strain calculations. If the phase change between the loading and the resulting flexural displacement is required then a dynamic model is necessary since it is clear from fig. 3.14 that the phase depends critically on wave period.

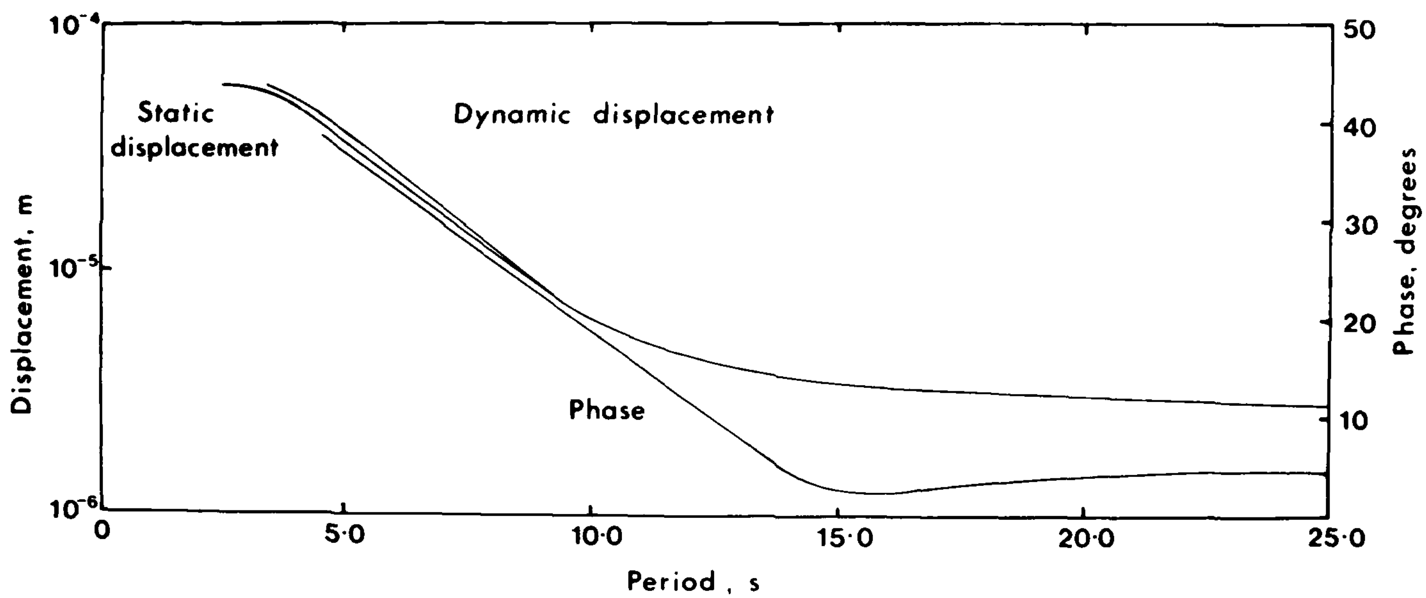


Fig. 3.14 Comparison of static and dynamic response for various wave periods.

4. PRELIMINARY COMPARISON OF THEORY WITH EAST GREENLAND DATA

4.1 Introduction

The previous sections have enabled us to compute theoretically all the parameters that have been measured in a series of wave experiments carried out by SPRI on sea ice in the Arctic. These results are reported or will be published elsewhere in great detail. See for example Squire and Moore (1980), Squire and Martin (1980), Goodman et al (1980), and Moore and Wadhams (1981). However, it was felt that no theoretical model could really be presented without some experimental verification.

The 1979 field programme in east Greenland had two principal aims: a) to quantify the motions of a single ice floe in waves; and b) to measure the attenuation of waves through pack ice in the fjord and to relate this to ice morphology. Several projects of type (a) were carried out and the author has chosen one of these experiments to test the proposed model. The comparisons are by no means exhaustive since suitable instrumentation to measure roll, pitch and yaw was not available during the field trip, and only a small subset of the recorded data is considered. Furthermore, the only data available for comparison at this time is not ideally suited as a test of the proposed model. The experiment we will discuss took place on 14 September over the entire day. Three accelerometers were used to measure heave, surge and sway, several strainmeters were used to measure the floe's surface strain field in various locations and directions, and the sea state was continuously monitored by means of a spar-loaded wave buoy. Since the field trip the accelerometer data have been analysed as random time series and have been integrated to give power spectra over half-hour segments throughout the day. By considering the spectrum of each of the floe-mounted instruments with that produced from the simultaneous wave buoy record, frequency response functions and coherences have also been produced which have enabled us to treat the motions of the ice floe independently from the incident wave forcing.

4.2 Results of Comparison

The ice floe upon which our experiments of 14 September took place was by no means ideal. The floe was approximately 80m to 90m across and between 3m and 4m thick. A 1m to 2m ridge adjacent to a refrozen meltpool ran along one side of the floe and cut off a small flat apron from our field of view. The instruments were deployed on a large area of suitably flat ice as far as possible from this ridge. For the purpose of the numerical simulation several possible floe geometries were coded and their effects on the resulting motions was studied in detail. For a floe of this size, it was found that an increase in diameter from 80m to 90m or a change in thickness from 3m to 4m, had little effect on the resulting motions. The ridge, however, did appear to influence the motions significantly, though in all cases the theoretical curves fitted the data better when a ridge and its corresponding keel were included in the geometry.

First attempts at a comparison between theory and experiment produced unsatisfactory results. There were clear discrepancies for both sway and strain, though the heave results looked promising. This can easily be explained when the location and drift of the ice floe are considered in relation to the shape and bathymetry of the fjord (fig. 4.1). Throughout the experiment the floe drifted seawards in deep water at about 1.7 km/hr, and always remained within a few hundred metres of a steep cliff face. Waves entering the fjord from the open ocean would experience two effects; firstly their spectrum would be distorted due to refraction, and secondly waves would reflect from the cliffs alongside the ice floe. Neglecting refraction, we are left with the considerable influence that an almost perfectly reflecting cliff could have on the motions of ice floes in its vicinity. For the sake of this argument we shall assume that the cliff is a *perfect reflector for all the wave periods present in the fjord.*

Consider a wavebuoy located in the fjord alongside the ice floe (fig. 4.2). This buoy will measure the superposition of both the incident

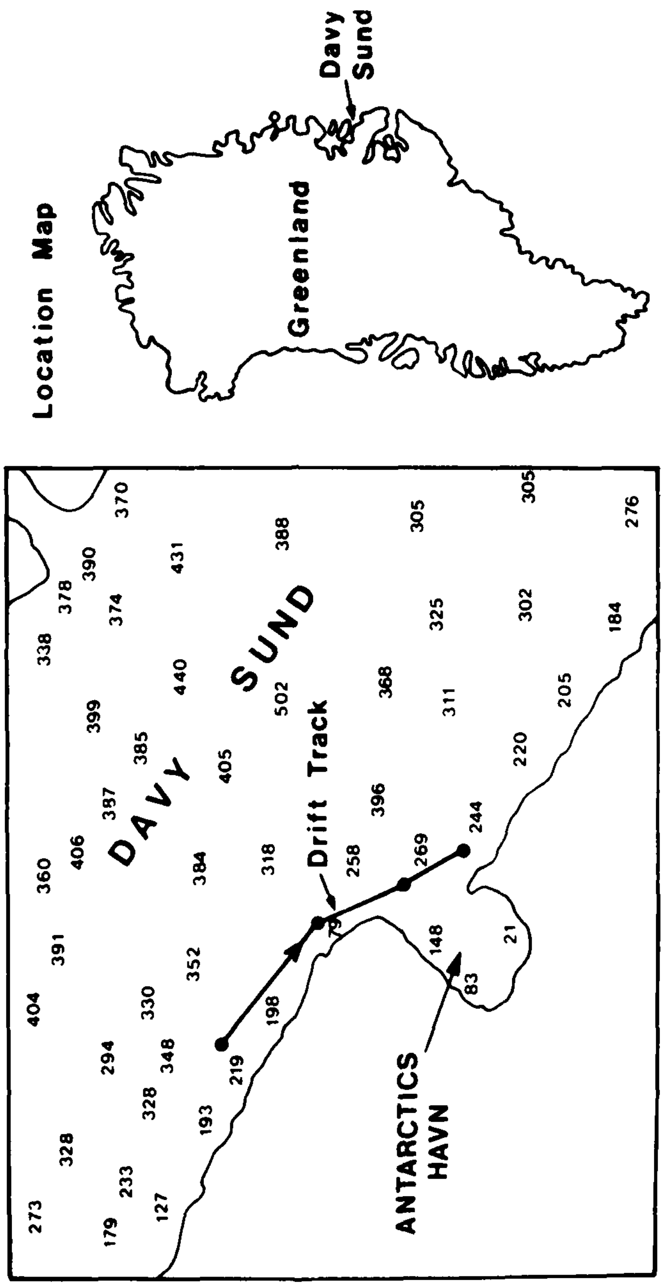


Fig. 4.1 Bathymetry and location of experimental site in East Greenland.

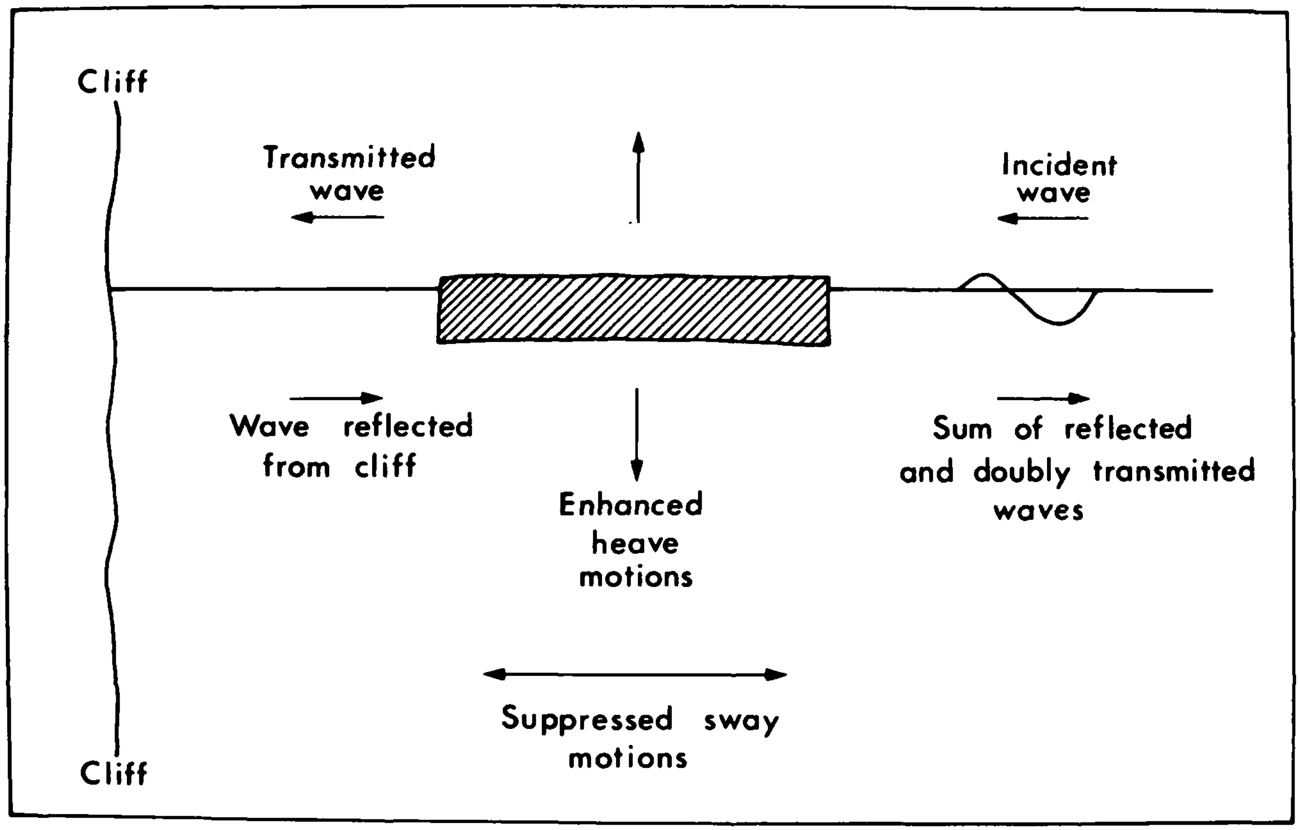


Fig. 4.2 The instrumented ice floe showing position of cliff and its effect on the heave and sway motions.

waves and the waves reflected back from the cliff face. When waves from any direction are considered, the sea sampled by the buoy becomes very complex. This is equally true when the motions of the floe itself are considered. If we consider the motions of heave, sway and strain in turn, we soon see that a comparison of the data with the numerical model is not possible except for the case of heave.

For heave, the two vertical accelerometers are used in the same sense, so that although the floe is behaving in a very complex fashion due to the incident waves, the normalisation carried out during the data processing is sufficient to enable a reasonable test of the model to be made. The heave data from five half-hour records are presented as a single frequency response function in fig. 4.3, where each experiment is distinguished by a different symbol. Confidence limits at the 95% level are also shown. The valid range (6s to 15s) marked in the figure represents the range of energy over which the original power spectra used to generate each plot had significant energy. Outside this range the gain factors have no physical meaning and are subject to large rounding error. Frequency smoothing across fifteen contiguous energy values has been used to generate each point. The smooth curve plotted on the data represents the gain factor predicted by theory. There appears to be excellent agreement between the model and experiment over the valid 6s to 15s range of periods. When a significant keel is included in the geometry of the numerical model, the theoretical curve changes little for low frequencies, but as we move toward higher frequencies so the curve moves up fractionally so as to centralize itself within the confidence limits.

The sway and surge data pose much more serious problems since the normalisation is no longer reasonable if the wavebuoy record is contaminated with wave energy reflected from the cliff face. Furthermore, if we think in terms of normal incidence for a moment, and suppose that the sea close to the floe is made up of waves propagating in two opposing directions, then the floe's sway motion will be considerably reduced. Thus the frequency response functions created by normalisation of the sway and surge records with respect to those of the wavebuoy will be very different from those computed for an open water situation. Unfortunately, it is very neary impossible to adjust either theory or observation, so that the sway

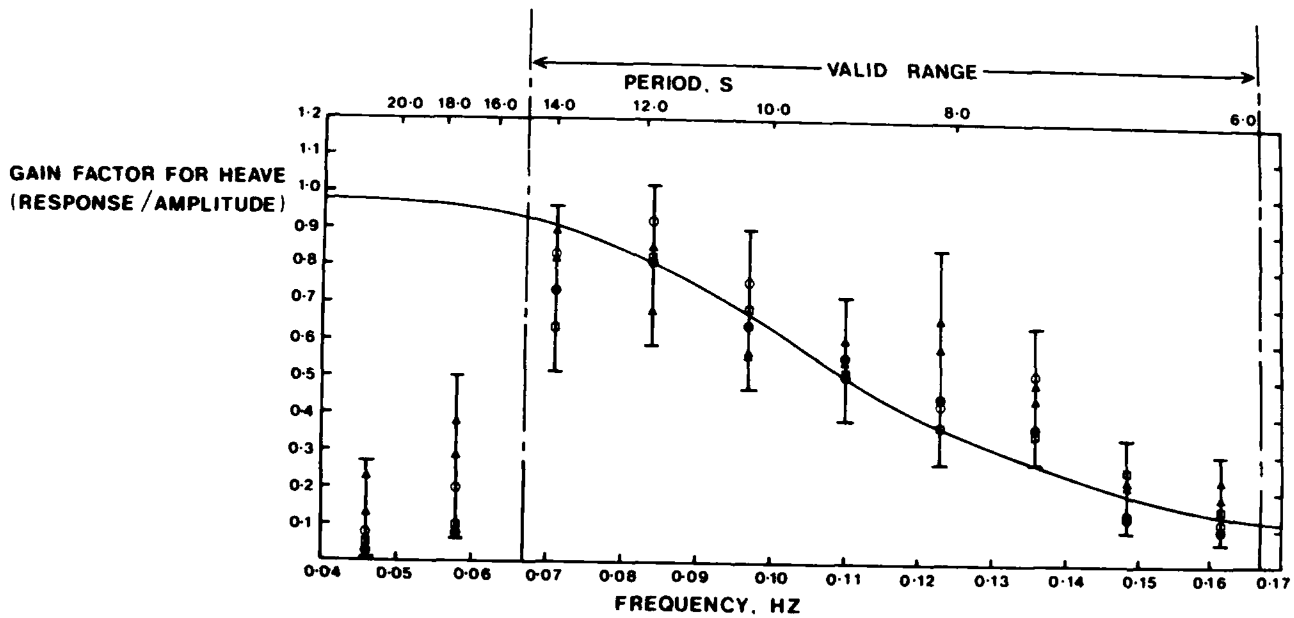


Fig. 4.3 Comparison of experimental and theoretical heave results for ice floe.

and surge results can only be discussed in a qualitative sense.

The gain factors of the frequency response functions for the combined horizontal motions show the same general shape as the theoretical curves with a resonance between 8s and 10s. However, the gain magnitudes are considerably smaller than the theory predicts as would be expected from the foregoing discussion. A detailed comparison must therefore await the processing of further east Greenland data recorded on floes which were further off the coast.

The strain data must also be subject to the same contamination due to waves reflected from the cliffs. As before, the ice floe will experience a wave load made up of both standing and travelling waves of a magnitude which depends on the wave's period. The reflection/transmission characteristics of the floe will determine its subsequent flexural behaviour. One would expect long waves to have small reflection coefficients so that most of the wave energy will reach the cliffs and be reflected back, whereas short waves would be considerably reduced in amplitude at the *initial reflection from the floe's front edge.* This leads one to tentatively suggest that short waves might be less sensitive to reflections from the cliffs, so that the theory would provide a better fit. This is indeed the case with theory and data being in good agreement for wave periods less than ten seconds, but becoming unacceptable as the wave period is increased.

At this stage then the author can present no better than an explanation why the data and model produce different results. Until more data are processed to produce the necessary frequency response functions for floes which are a reasonable distance from the edge of the fjord, no detailed theory can model the ice floe's motions. This analysis promises to take at least two months from the date of publication of this article due to the complex problems which arise when strainmeter data are processed. A later paper to be published in the literature will provide a more convincing verification of the proposed numerical model.

5. BIBLIOGRAPHY

Bendat, J.S., and A.G. Piersol. 1971. Random Data: Analysis and Measurement Procedures, Wiley-Interscience, New York, 407pp.

Desai, C.S., and J.F. Abel. 1972. Introduction to the Finite Element Method. Van Nostrand Reinhold Co., 477pp.

Frank, W. 1967. Oscillation of cylinders in or below the free surface of deep fluids. NSRDC Rep. 2375, Naval Ship Research and Development Center, Washington D.C., U.S., 40pp.

Goodman, D.J., Wadhams, P., and V.A. Squire. 1980. The flexural response of a tabular ice island to ocean swell. Annals of Glaciology, 1, 23-27.

Hendrickson, J.A. 1966. Interaction theory for a floating elastic sheet of finite length with gravity waves in water of finite depth. Rep. NBy-62185 by Natn. Engng. Science Co., Pasadena, Calif. for U.S. Naval Engng. Lab., Port Hueneme, Calif., 178pp.

Jaeger, J.C. 1956. Elasticity, Fracture and flow with Engineering and Geological Applications. Methuen and Co., Ltd., New York, 152pp.

John, F. 1950. On the motion of floating bodies II, Comm. Pure. Appl. Math., 3, 45-101.

Lamb, H. 1962. Hydrodynamics, Cambridge University Press, 6th ed., 738pp.

Lee, C.M. 1976. Motion characteristics of floating bodies. J. Ship Res., 20(4), 181-189.

McCormick, M.E. 1973. Ocean Engineering Wave Mechanics, Wiley-Interscience, John Wiley and Sons, Lond., 179pp.

Moore, S.C., and P. Wadhams. 1981. Recent developments in strainmeter design. Proceedings of the Workshop on Sea Ice Field Measurement, St. Johns, Newfoundland, April 29-May 1, 1980, 99-125.

Robin, G.de Q. 1963. Wave propagation through fields of pack ice. Phil. Trans. R. Soc. A, 255(1057), 313-339.

Squire, V.A. 1978. Dynamics of Ocean Waves in a Continuous Sea Ice Cover, Ph.D. dissertation, Univ. of Cambridge, England, 191pp.

Squire, V.A., and S. Martin. 1980. A field study of the physical properites, response to swell, and subsequent fracture of a single ice floe in the winter Bering Sea. Scientific Rep. 18, Dept. of Atmospheric Sciences and Oceanography, Univ. of Washington, U.S. 56pp.

Squire, V.A., and S.C. Moore. 1980. Direct measurement of the attenuation of ocean waves by pack ice. Nature Lond., 283(5745), 365-368.

Timoshenko, S., and S. Woinowsy-Krieger. 1959. Theory of Plates and Shells, McGraw-Hill Book Co., New York, 2nd ed., 580pp.

Timoshenko, S., Young, D.H., and W. Weaver Jr. 1974. Vibrational Problems in Engineering, John Wiley and Sons, New York, 4th ed., 521pp.

Ursell, F. 1949a. On the heaving motion of a circular cylinder on the surface of a fluid. Q. Jl. Mech. appl. Math., 2, 218-231.

Ursell, F. 1949b. On the rolling motion of cylinders in the surface of a fluid. Q. Jl. Mech. appl. Math., 2, 335-353.

Vugts, J.H. 1968a. The hydrodynamic coefficients for swaying, heaving and rolling cylinders in a free surface. Rep. No. 112S, Nederlands Scheepsstudiecentrum Tno, Leeghwaterstraat 5, Delft, Netherlands, 31pp.

Vugts, J.H. 1968b. Cylinder motions in beam waves. Rep. No. 115S, Nederlands Scheepsstudiecentrum Tno, Leeghwaterstraat 5, Delft, Netherlands, 23pp.

Wadhams, P. 1973a. Attenuation of swell by sea ice. J. Geophys. Res., 78(18), 3552-3563.

Wadhams, P. 1973b. The Effect of a Sea Ice Cover on Ocean Surface Waves, Ph.D. dissertation, Univ. of Cambridge, England, 223pp.

Wadhams, P. 1978. Wave decay in the marginal ice zone measured from a submarine. Deep Sea Res., 25, 23-40.

Wehausen, J.V. 1971. The motion of floating bodies. Annual Review of Fluid Mechanics, 3, 237-268.

Wehausen, J.V., and E.V. Laitone. 1960. Surface Waves. Encyclopeia of Physics, Vol. IX, Springer-Verlag, Berlin, 446-778.

Zienkiewicz, O.C. 1977. The Finite Element Method, McGraw-Hill Book Co. (UK) Ltd., 3rd ed., 787pp.

DATE
FILMED
-8